Introduction to Soil Mechanics and Shear Strength

Learning Objectives

- Understand different soil types.
- Understand the basics of soil shear strength.
- Understand what conditions dictate the type of shear strength that should be considered.

Definition

- Soil – an un-cemented aggregate of rock and mineral grains and decaying organic matter (solid particles) with liquid and/or gas occupying the void space between the solid particles.

Properties of Solid Particles

- Particle size

Small Large

Clay -- Silt -- Sand -- Gravel -- Cobbles -- Boulders

0.075mm	4.75 mm	75mm	350mm

A quantitative measure of soil particles.

- Particle shape

Rounded -- Subrounded -- Subangular -- Angular

A qualitative measure of soil particles.

Engineering Soil Classification

- The purpose:
  - Common language for naming soils.
  - Mixtures of particles that vary in mineralogy and particle size.
  - Categorize soils based on their engineering properties and characteristics.
  - Based partly on grain size.
  - Based partly on index properties.

Silts and Clays

- Particles smaller than we can detect by visual or sieve analysis methods.
- Silts are very fine bulky particles like sands – they are rock and mineral fragments – they are reasonably inert.
- Clays are minerals that are the product of chemical weathering of feldspar, ferromagnesian, and mica minerals.
- Clay particles are very small and are chemically active.
- Silts and Clays exhibit different engineering behavior.

TABLE 1 Soil Classification Chart

Criteria for Assigning Group Symbols and Group Names Using Laboratory Tests[A]				Soil Classification: Group Symbol	Soil Classification: Group Name[B]
Coarse-Grained Soils More than 50 % retained on No. 200 sieve	Gravels More than 50 % of coarse fraction retained on No. 4 sieve	Clean Gravels Less than 5 % fines[C]	Cu ≥ 4 and 1 ≤ Cc ≤ 3[E]	GW	Well-graded gravel[F]
			Cu < 4 and/or 1 > Cc > 3[E]	GP	Poorly graded gravel[F]
		Gravels with Fines More than 12 % fines[C]	Fines classify as ML or MH	GM	Silty gravel[F,G,H]
			Fines classify as CL or CH	GC	Clayey gravel[F,G,H]
	Sands 50 % or more of coarse fraction passes No. 4 sieve	Clean Sands Less than 5 % fines[D]	Cu ≥ 6 and 1 ≤ Cc ≤ 3[E]	SW	Well-graded sand
			Cu < 6 and/or 1 > Cc > 3[E]	SP	Poorly graded sand[I]
		Sands with Fines More than 12 % fines[D]	Fines classify as ML or MH	SM	Silty sand[G,H,J]
			Fines classify as CL or CH	SC	Clayey sand[G,H,I]
Fine-Grained Soils 50 % or more passes the No. 200 sieve	Silts and Clays Liquid limit less than 50	inorganic	PI > 7 and plots on or above "A" line[J]	CL	Lean clay[K,L,M]
			PI < 4 or plots below "A" line[J]	ML	Silt[K,L,M]
		organic	$\frac{\text{Liquid limit – oven dried}}{\text{Liquid limit – not dried}} < 0.75$	OL	Organic clay[K,L,M,N] / Organic silt[K,L,M,O]
	Silts and Clays Liquid limit 50 or more	inorganic	PI plots on or above "A" line	CH	Fat clay[K,L,M]
			PI plots below "A" line	MH	Elastic silt[K,L,M]
		organic	$\frac{\text{Liquid limit – oven dried}}{\text{Liquid limit – not dried}} < 0.75$	OH	Organic clay[K,L,M,P] / Organic silt[K,L,M,Q]
ghly organic soils		Primarily organic matter, dark in color, and organic odor		PT	Peat

[A] Based on the material passing the 3-in. (75-mm) sieve.

[B] If field sample contained cobbles or boulders, or both, add "with cobbles or boulders, or both" to group name.

[C] Gravels with 5 to 12 % fines require dual symbols:
GW-GM well-graded gravel with silt
GW-GC well-graded gravel with clay
GP-GM poorly graded gravel with silt
GP-GC poorly graded gravel with clay

[D] Sands with 5 to 12 % fines require dual symbols:
SW-SM well-graded sand with silt
SW-SC well-graded sand with clay
SP-SM poorly graded sand with silt
SP-SC poorly graded sand with clay

[E] $Cu = D_{60}/D_{10}$ $\frac{(D_{30})^2}{D_{10} \times D_{60}}$

[F] If soil contains ≥ 15 % sand, add "with sand" to group name.

[G] If fines classify as CL-ML, use dual symbol GC-GM, or SC-SM.

[H] If fines are organic, add "with organic fines" to group name.

[I] If soil contains ≥ 15 % gravel, add "with gravel" to group name.

[J] If Atterberg limits plot in hatched area, soil is a CL-ML, silty clay.

[K] If soil contains 15 to 29 % plus No. 200, add "with sand" or "with gravel," whichever is predominant.

[L] If soil contains ≥ 30 % plus No. 200, predominantly sand, add "sandy" to group name.

[M] If soil contains ≥ 30 % plus No. 200, predominantly gravel, add "gravelly" to group name.

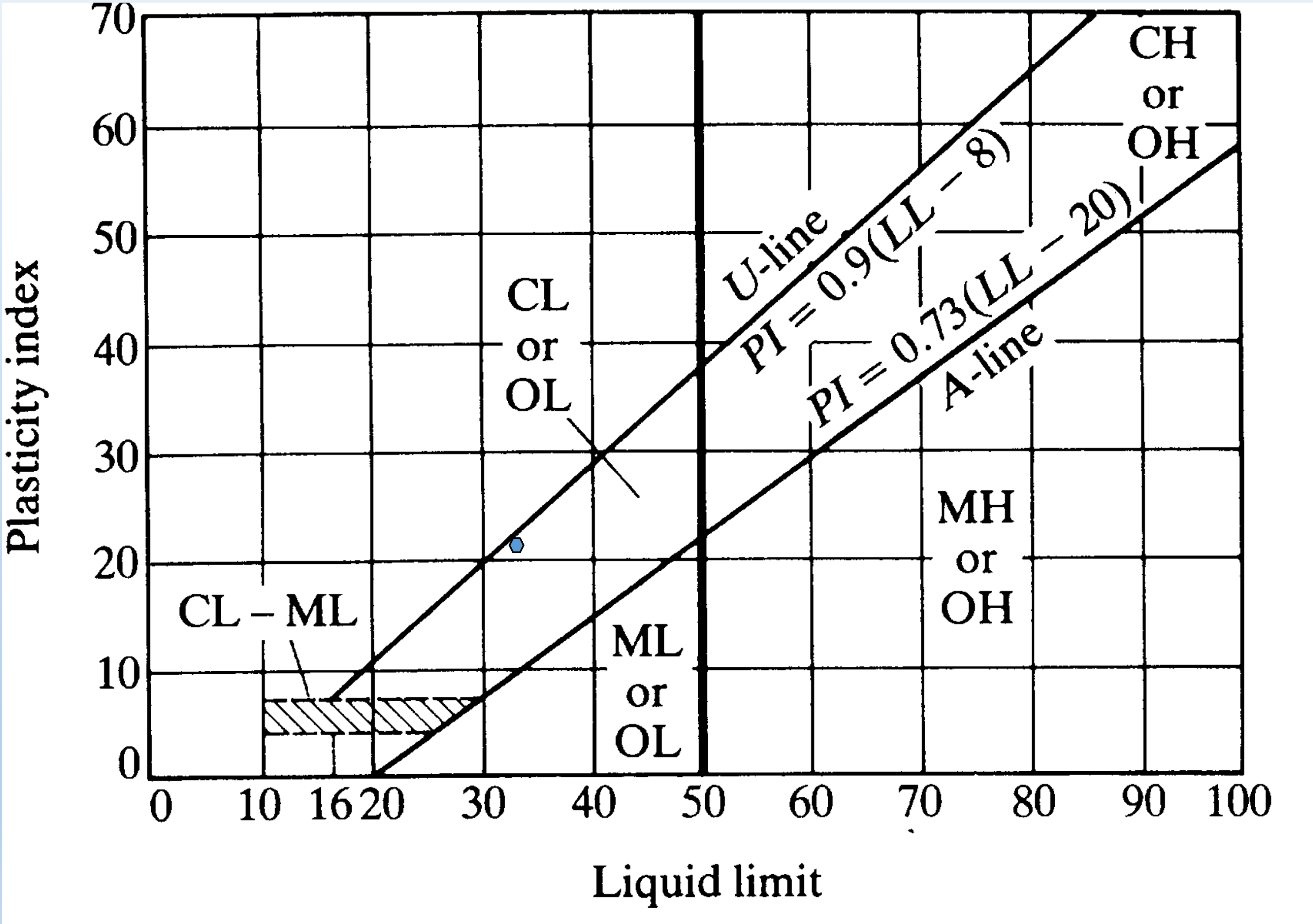
[N] PI ≥ 4 and plots on or above "A" line.

[O] PI < 4 or plots below "A" line.

[P] PI plots on or above "A" line.

[Q] PI plots below "A" line.

The Plasticity Chart



How do we determine shear strength of soil and rock?

- Direct Shear
- Triaxial Shear
- In-situ testing (Vane Shear, CPT, SPT correlations, Schmidt Hammer)
- Correlations
- Back-analysis of existing landslide geometry.

Most Important Concept: Effective Stress

- Effective Stress is arguably the most important concept in soil mechanics.
- It dictates the relationship between water pressure and the mobilized stress in a soil matrix.
- Most simply, it is defined as:

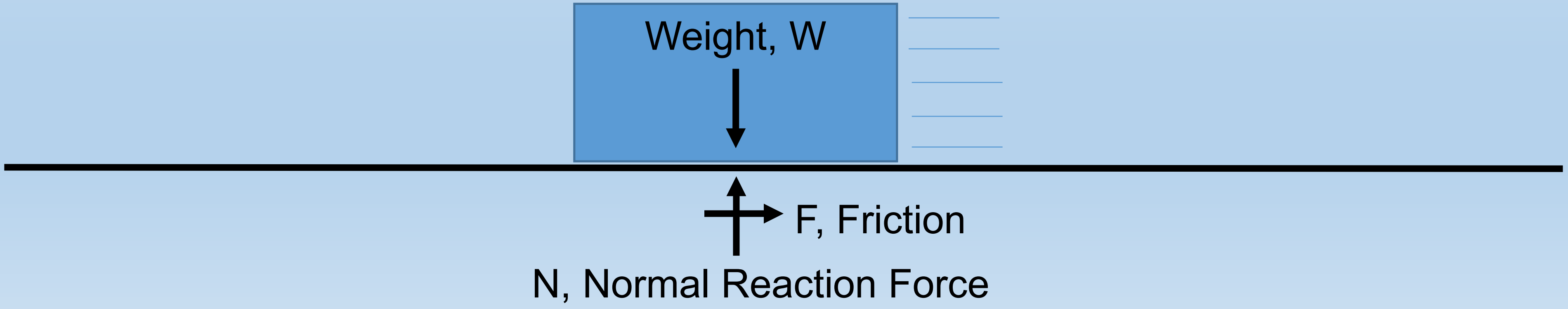
Effective Stress = Total Stress – Pore Water Pressure

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - \boldsymbol{u}$$

- Effective Stress = Actual Contact Forces between Soil Grains
- Total Stress = The total weight of soil and water within a column.
- Pore Water Pressure = The buoyant forces pushing grains apart.

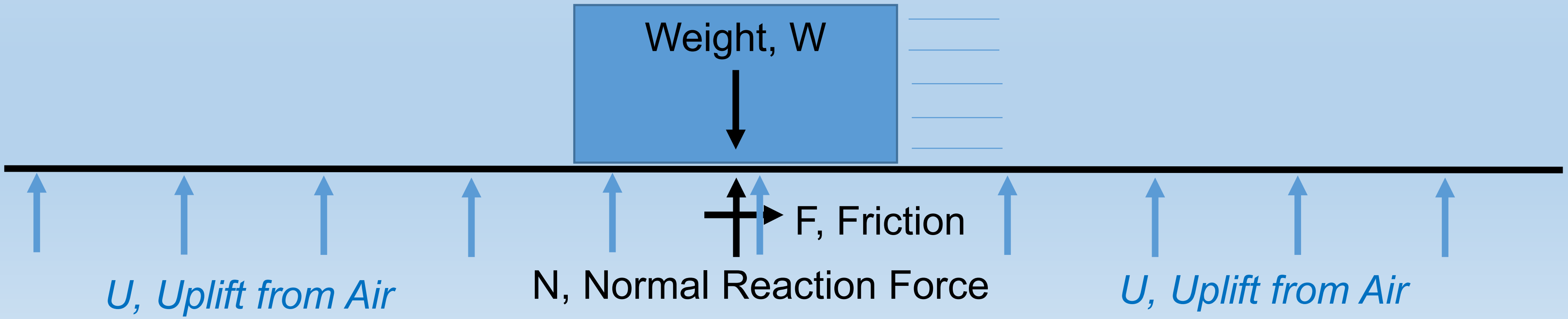
Concept of Effective Stress

- Let's take an example that we all remember from childhood: air hockey.
- When the air is off, the puck doesn't slide as well.
- The reaction force, N, is equal to the weight of the puck.
- Friction is N multiplied by a friction coefficient.



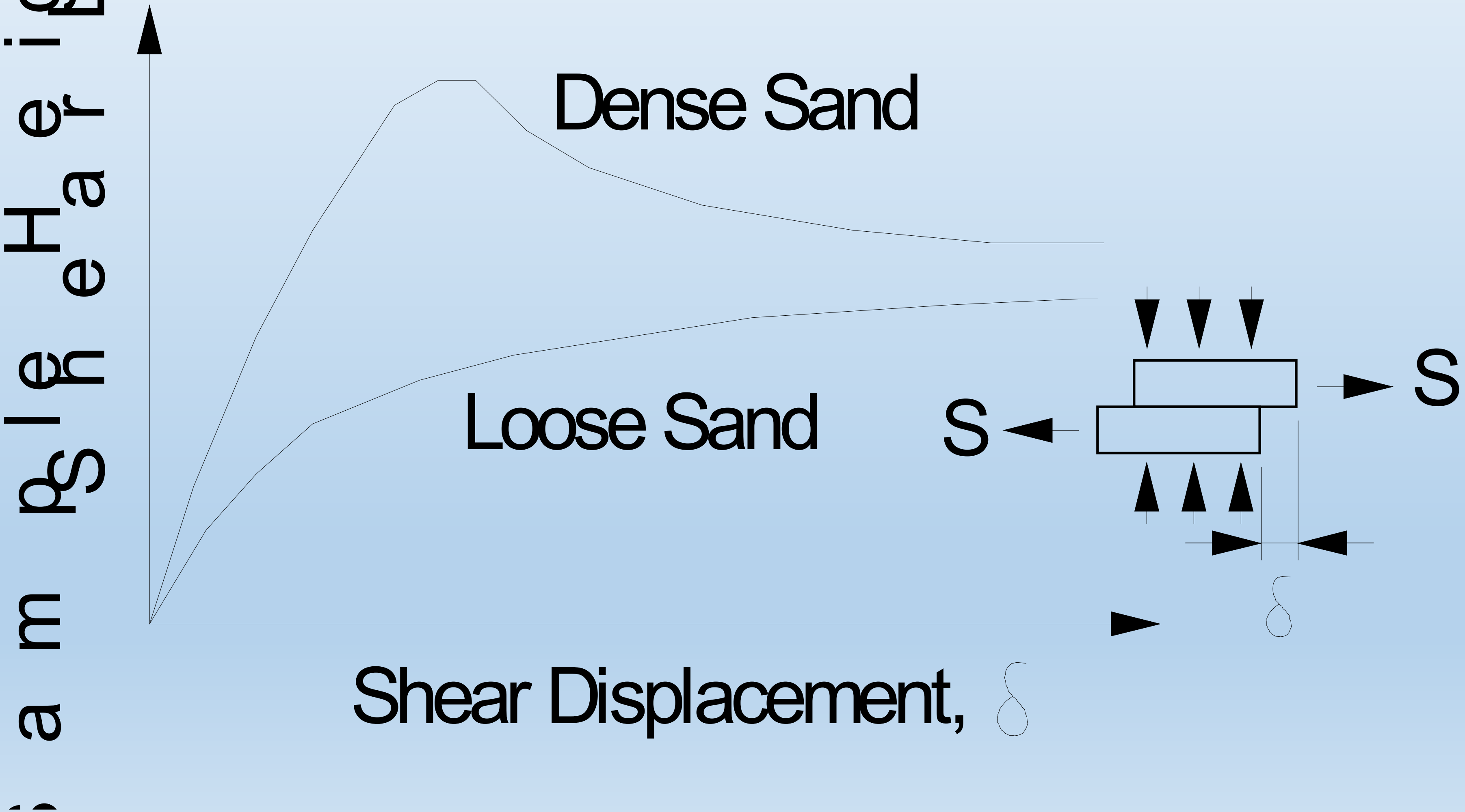
Concept of Effective Stress

- If we turn on air on the table, the reaction force is no longer equal to the weight.
- The normal decreases the by uplift force from air.
- Thus, friction decreases and the puck slides easily.
- This is conceptually the same as “buoyancy.”

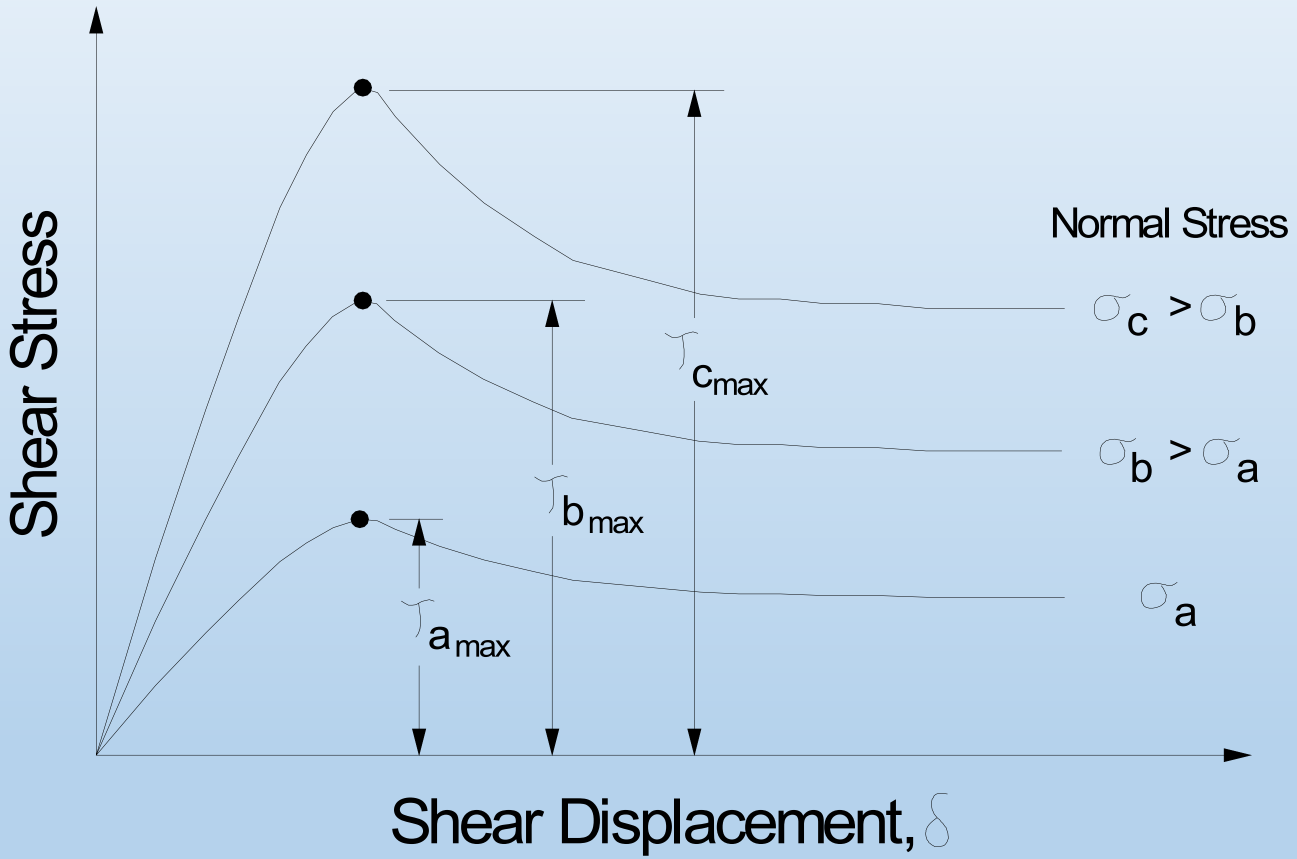


Shear Strength of Coarse-Grained Materials

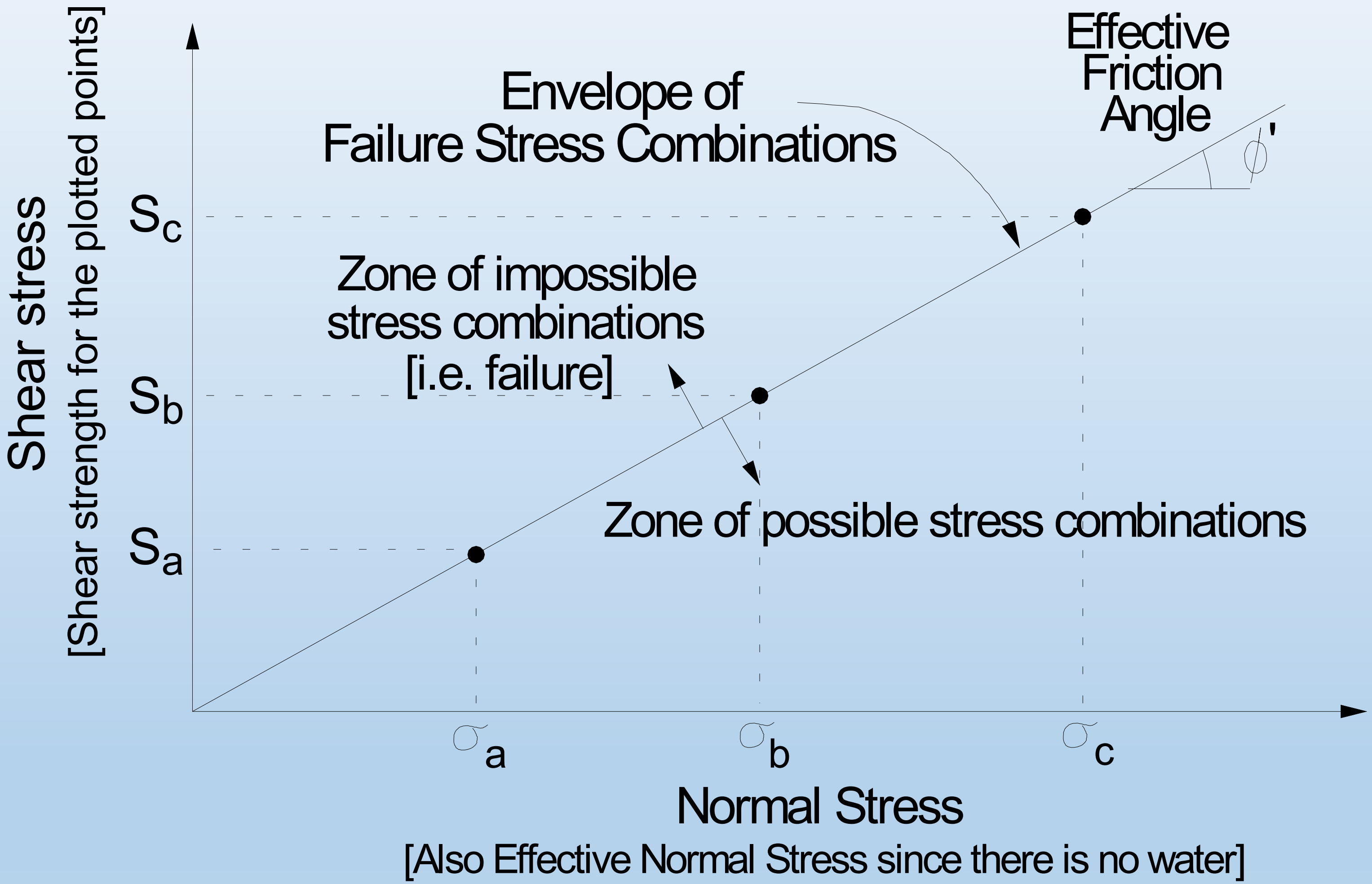
Direct Shear Tests on Dry Sand



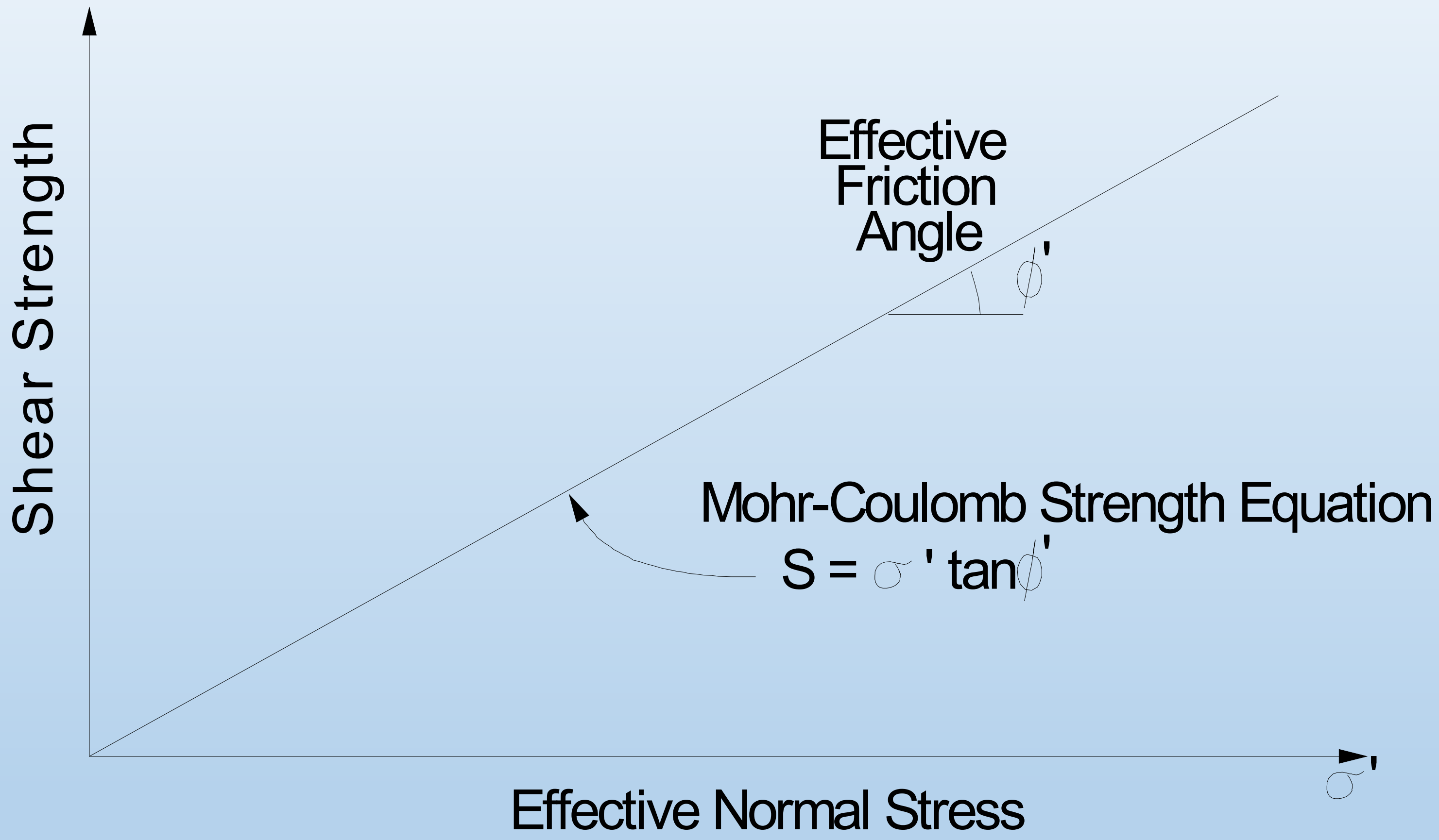
Effect of Normal Stress



Plot of shear strength versus normal stress



We can represent the strength as related to effective normal stress.



$$S = \sigma' \tan(\phi')$$

Shear Strength of Fine-Grained Materials

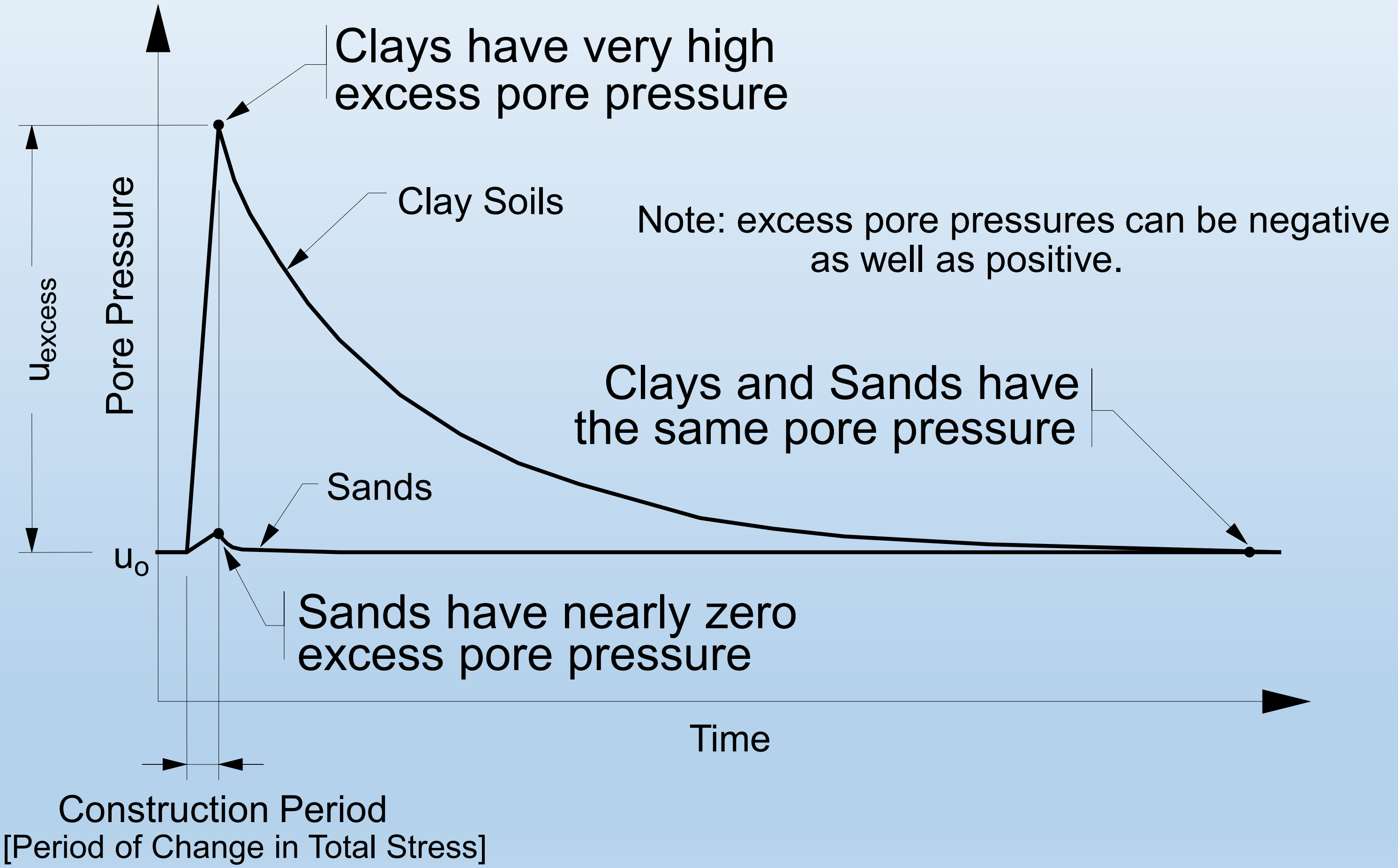
Similarities to Coarse-Grained Materials

- Shear strength is a function of effective stress.
- Shear strength is defined by the Mohr-Coulomb strength equation in terms of effective stress.
- In some cases the Mohr-Coulomb failure envelope [the line defined by the equation] passes through the origin.

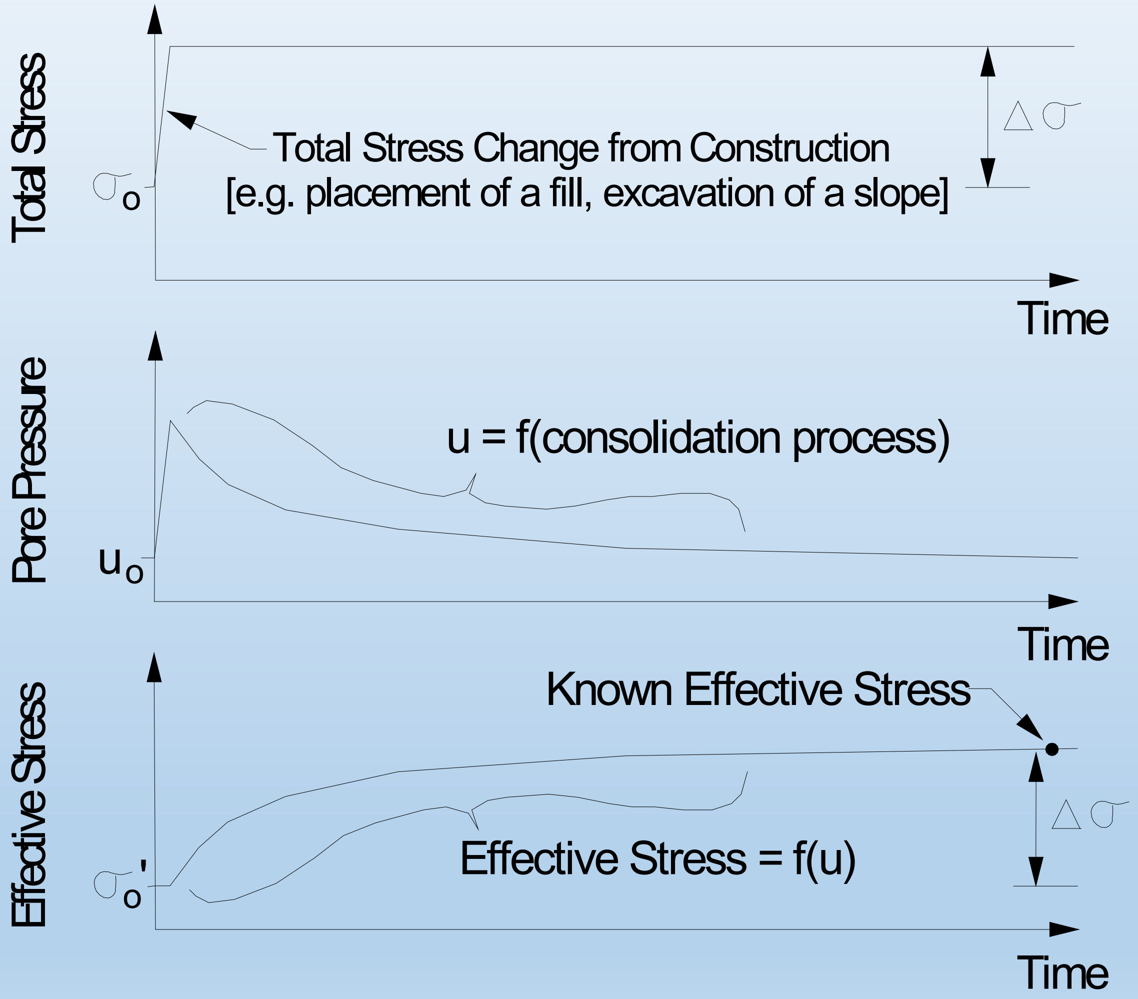
Differences

- In some cases the Mohr-Coulomb failure envelope will have a shear stress intercept -- it won't pass through the origin.
- It is important to account for how pore water pressures change with time.

Pore pressure and drainage response of soils



If we put the pore pressure behavior together with effective stress, we get:

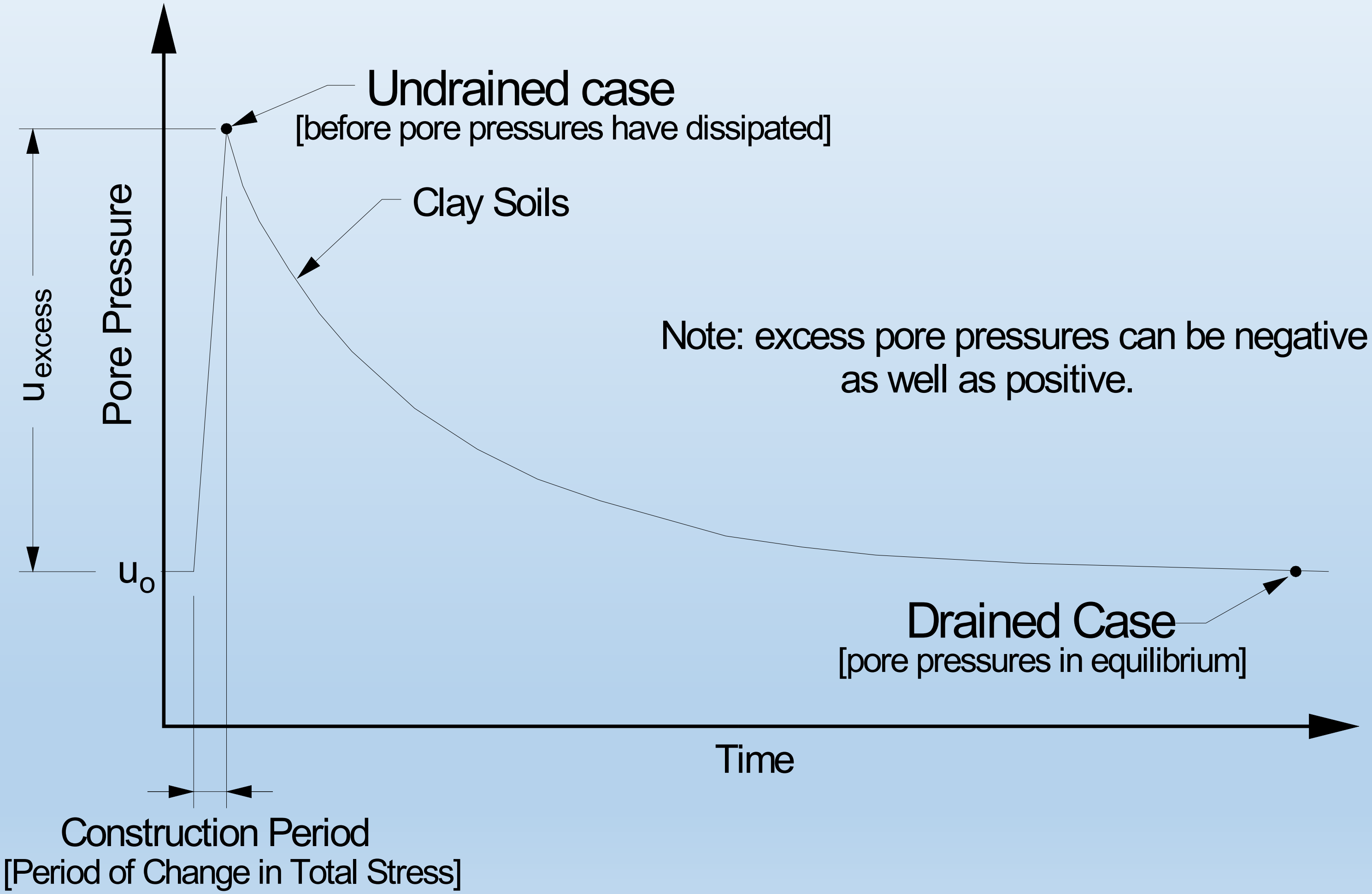


But there is a problem!

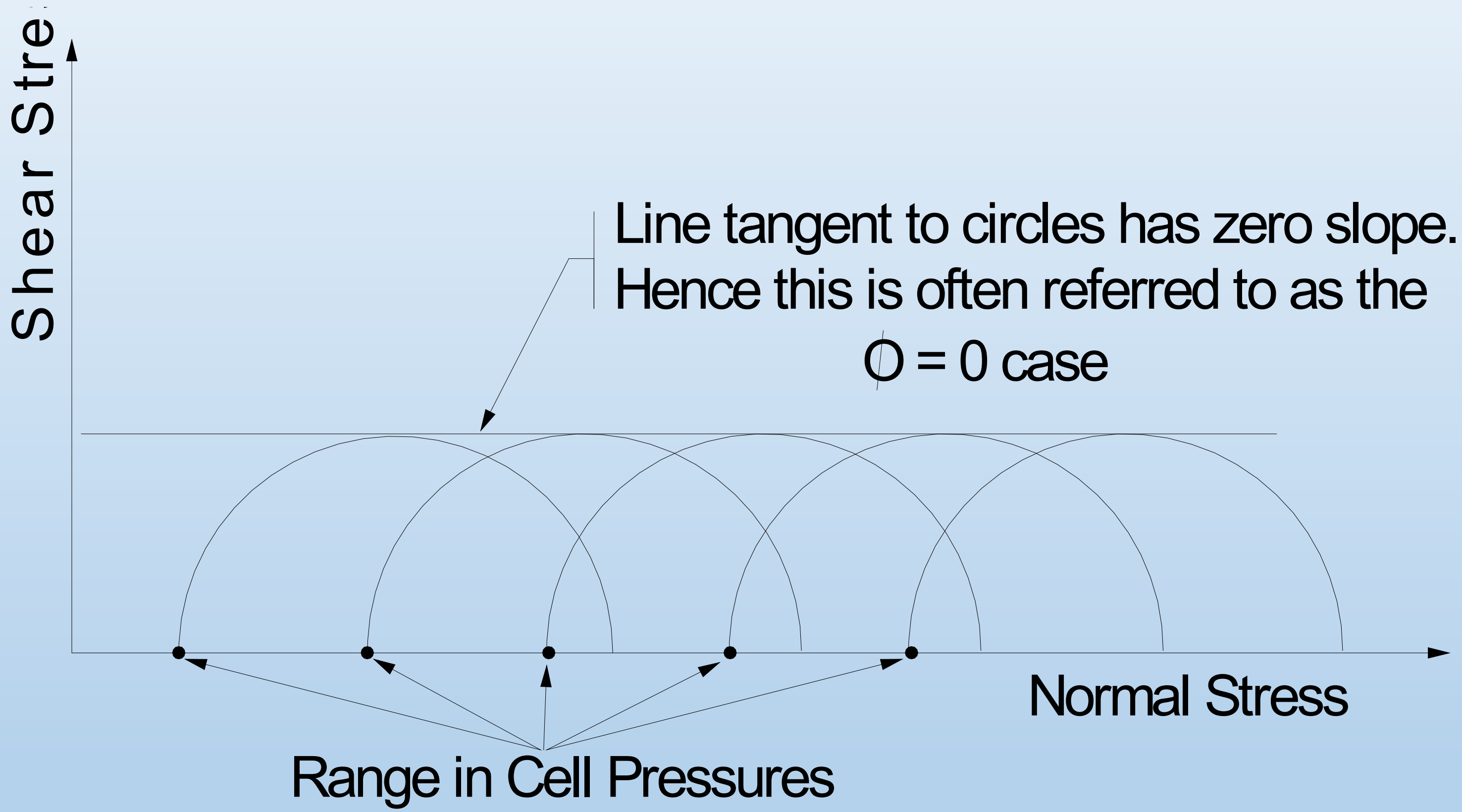
Drained versus Undrained Behavior

- Look only at the extreme cases.
- The “long term” or equilibrium case is the same as sand – the pore pressures are equal to the ambient values which are a function of the ground water table. This is called the Drained Case.
- The case immediately following loading, is termed the Undrained Case since little of the pore pressure generated in response to the Total Stress change would have disappeared.

The Undrained and Drained Strength States
-- illustrated in terms of pore pressure.

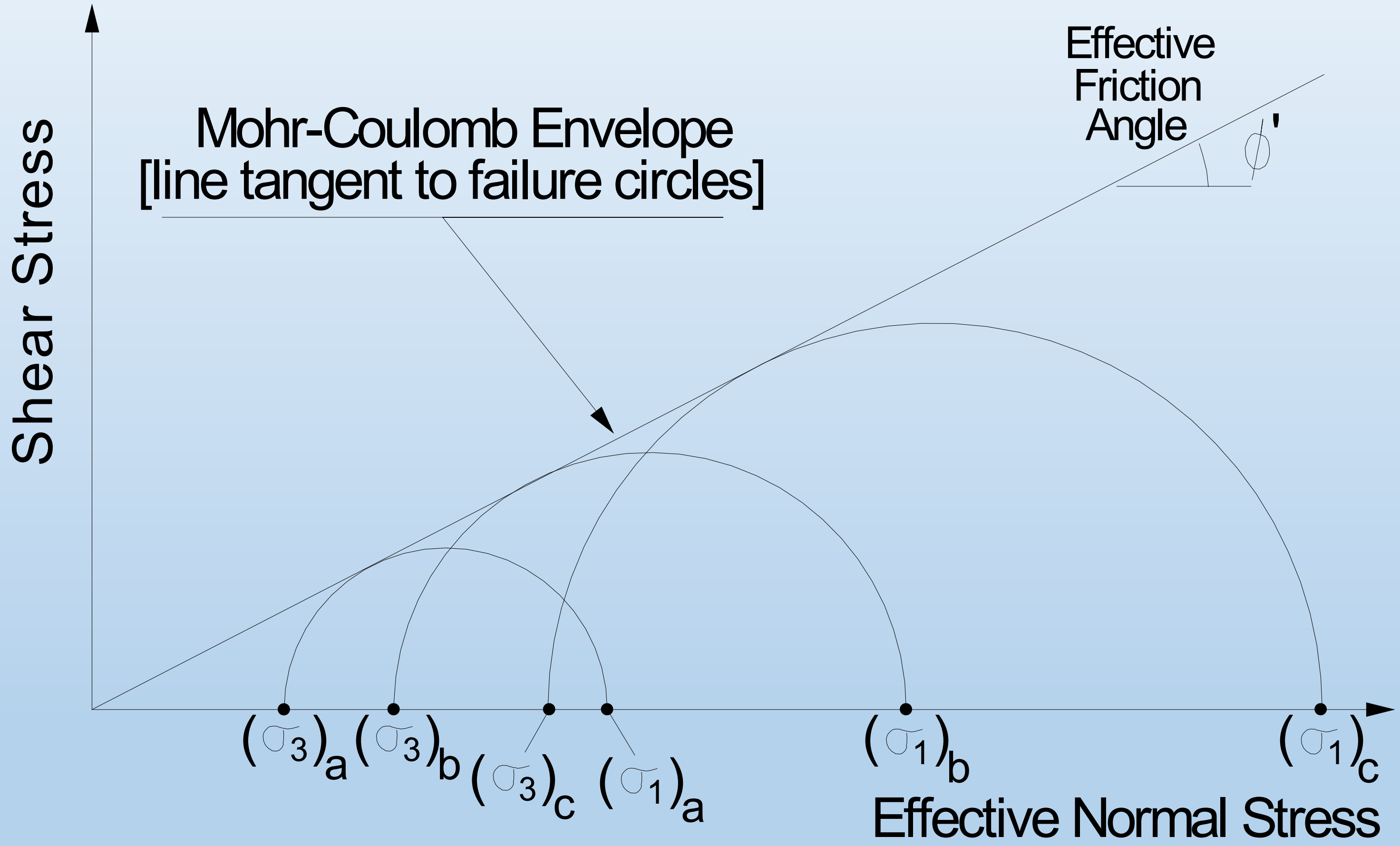


Undrained Shear Strength of Fine-grained Soils.



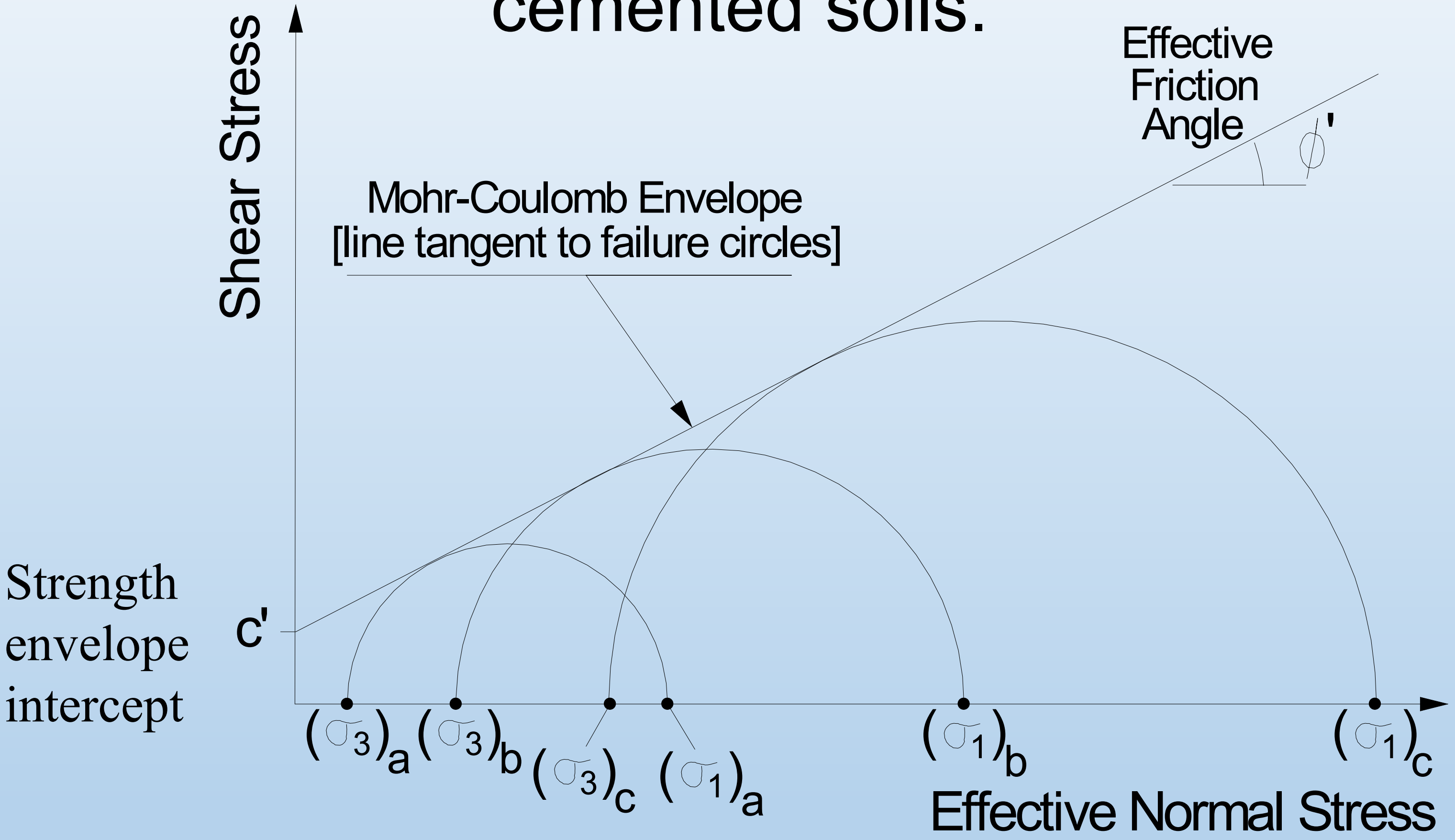
$$S = c_u$$

Typical drained shear strength for loosely-deposited fine-grained soil.



$$S = \sigma' \tan(\phi')$$

Typical drained shear strength for overconsolidated fine-grained soils or cemented soils.



$$S = c' + \sigma' \tan(\phi')$$

Total Stress, Effective Stress and Shear Strength Parameters

- Drained field conditions critical: Use effective stress analysis + drained shear strength parameters

- Undrained field conditions critical: Use total stress analysis + undrained shear strength parameters

- If in doubt, analyze twice each time using consistent shear strength parameters

Total Stress vs. Effective Stress in Stability Analysis

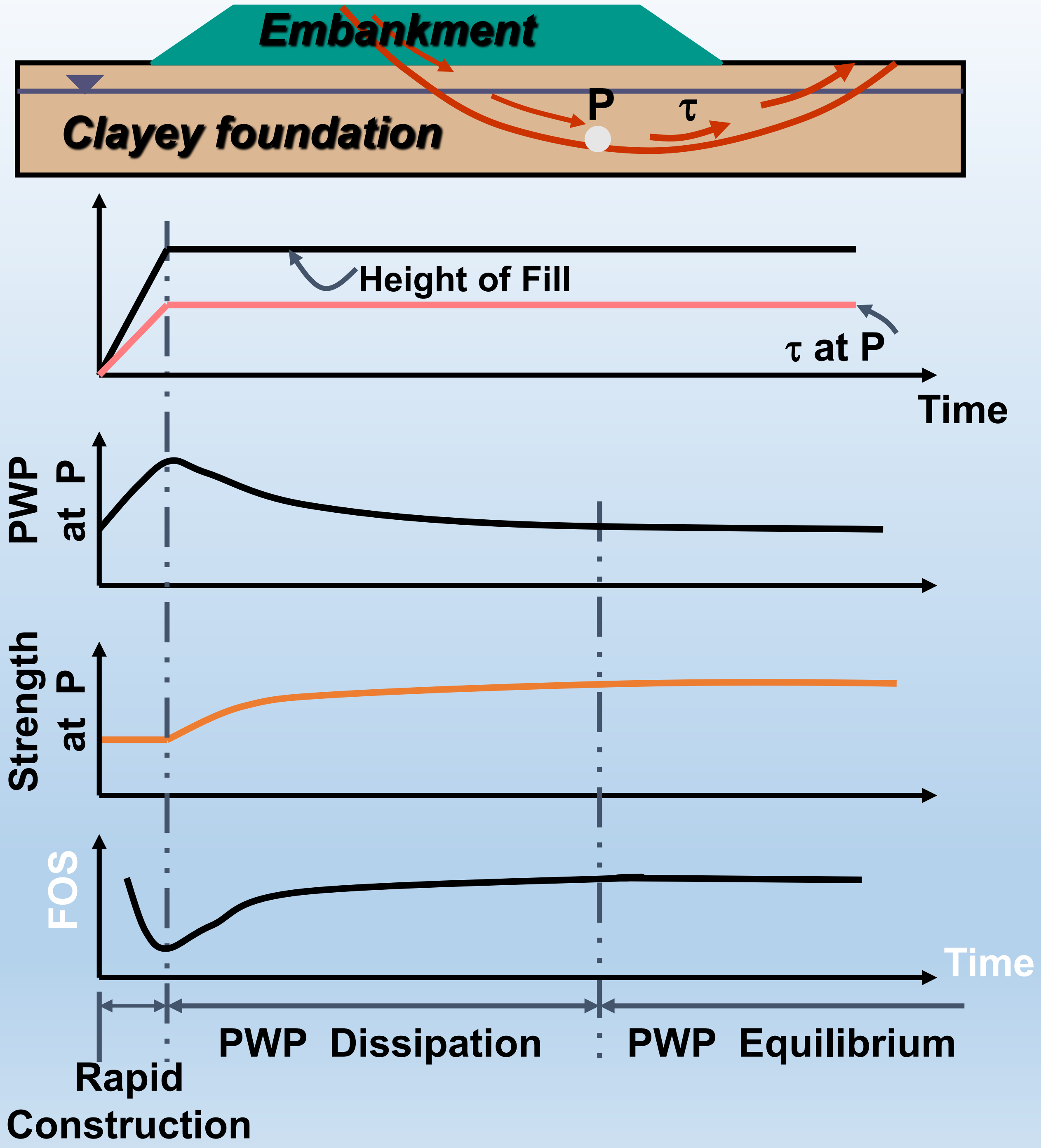
- Total stress analysis:
  - Only total soil stresses are used
  - PWP is not considered
  - Fast loading (construction, traffic, seismic)
- Effective stress analysis:
  - Only effective soil stresses are used
  - PWP must be considered
  - Slow loading (rainfall, slow changes, after construction)

Conceptual Process in Stability Analysis

1. Consider loading versus time
2. Consider PWP versus time
3. Consider shear resistance versus time
4. Assess the available resistance over driving load versus time $\rightarrow$ establish critical situation
5. Conclude with the type of stress and strength parameters to use in stability analysis

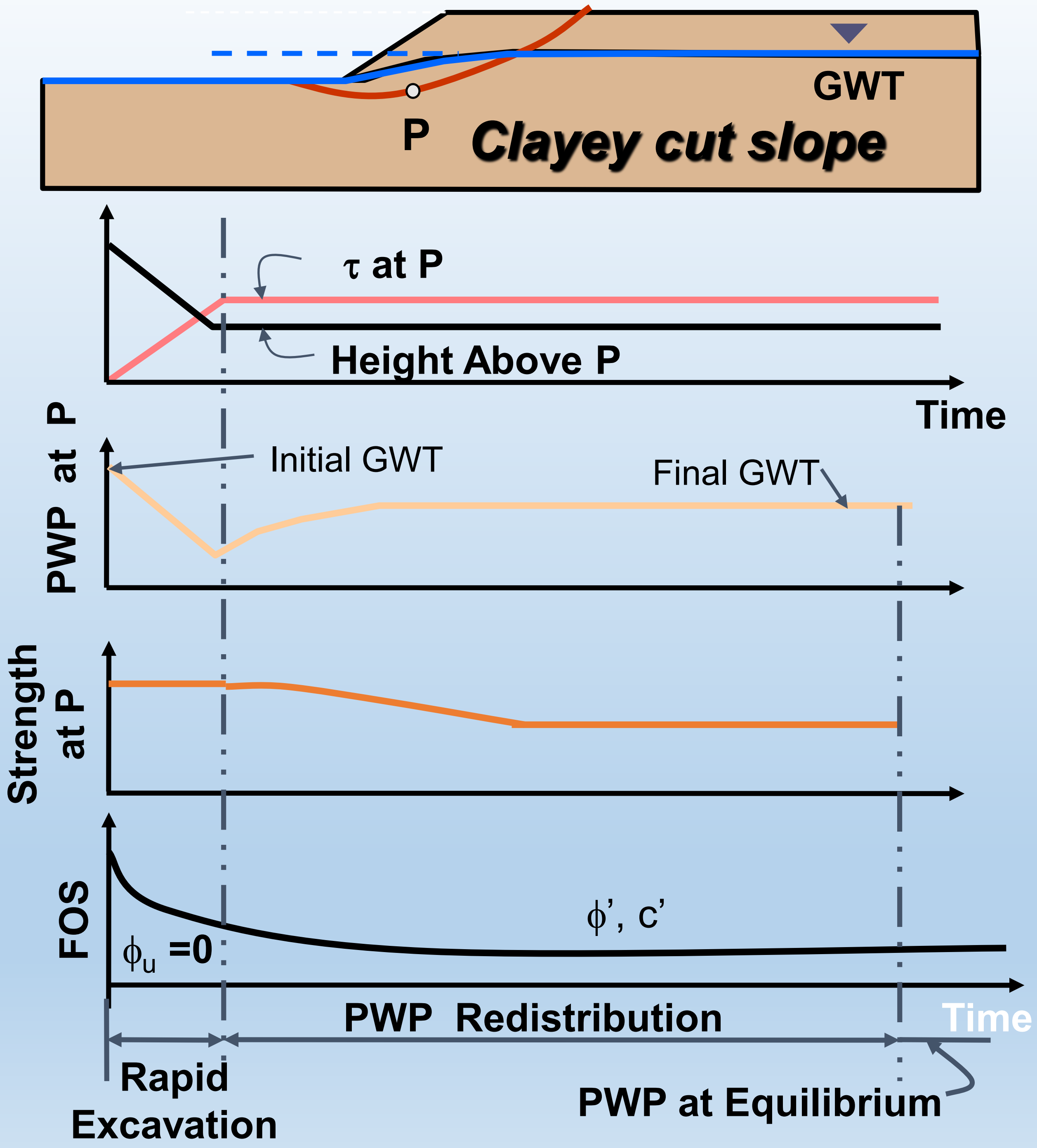
Justify these conclusions:

1. ***End of construction is critical***
2. ***Total stress analysis is appropriate***
3. ***Use un-drained shear strength parameters (ϕ_u=0)***



Justify these conclusions:

1. ***Long-term stability is critical***
2. ***Effective stress analysis is appropriate***
3. ***Drained shear strength parameters (ϕ' and c')***

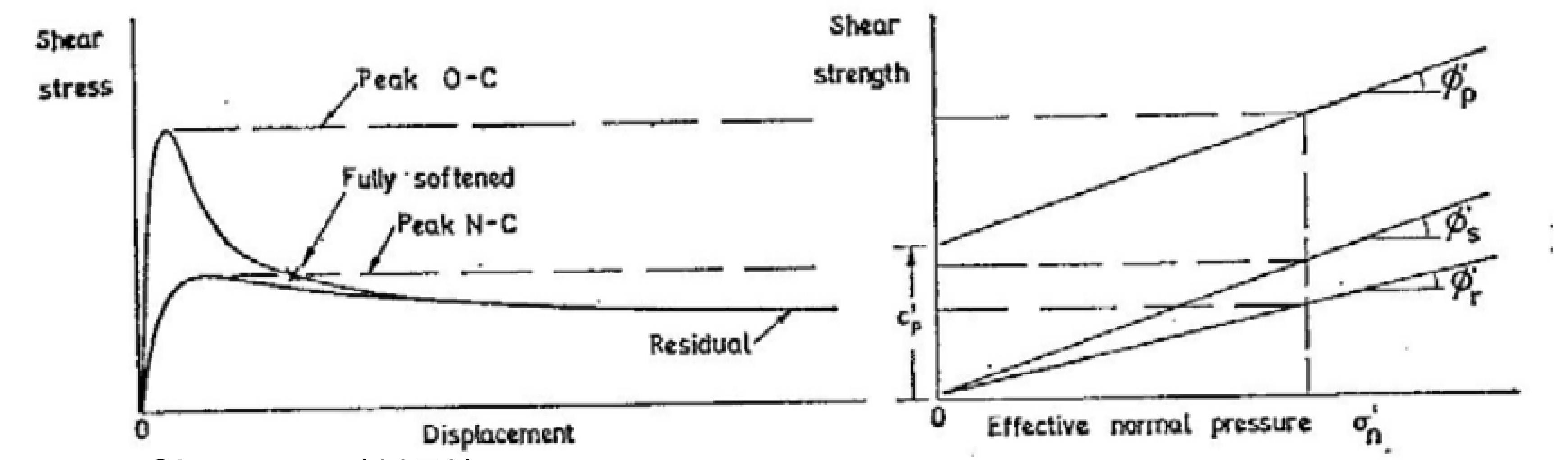


Summary

Aspect	*End of Construction*	*Multi-stage Loading*	*Long-term*
Analysis/Strength Free Draining	σ', ϕ', c'	σ', ϕ', c'	σ', ϕ', c'
Analysis/Strength Impermeable	σ, (ϕ_u=0, c_u) or (ϕ, c) → UU, CU	σ, (ϕ_u=0, c_u) → CU	σ', ϕ', c'
Pore Water Pressure	u=0 → total u≥0 → effective	u=0 → total u≥0 → effective	u≥0 → effective

The Residual State

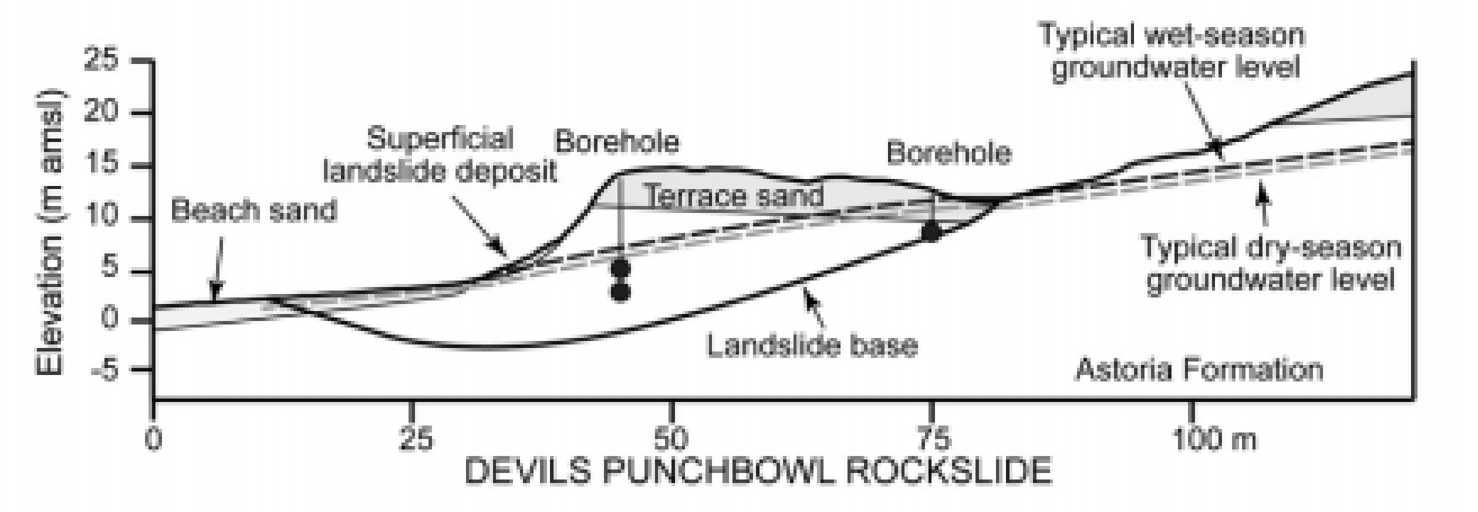
- When soil has sheared significantly (i.e. active landslide terrain), the grains align and cemented bonds are broken.
- Often, the cementation is what provides “cohesion” – this disappears.
- When the grains align, significantly less friction is mobilized.
- This occurs well-past peak shear strength…



Skempton (1970)

Why it matters.

- Active landslide on Oregon Coast (Schulz et al. 2007).
- Shear strength properties of undisturbed, cemented material:
  - Cohesion: 2340 psf
  - Friction: 26.7°
- Residual State:
  - Cohesion: 0 psf
  - Friction: 15.8°



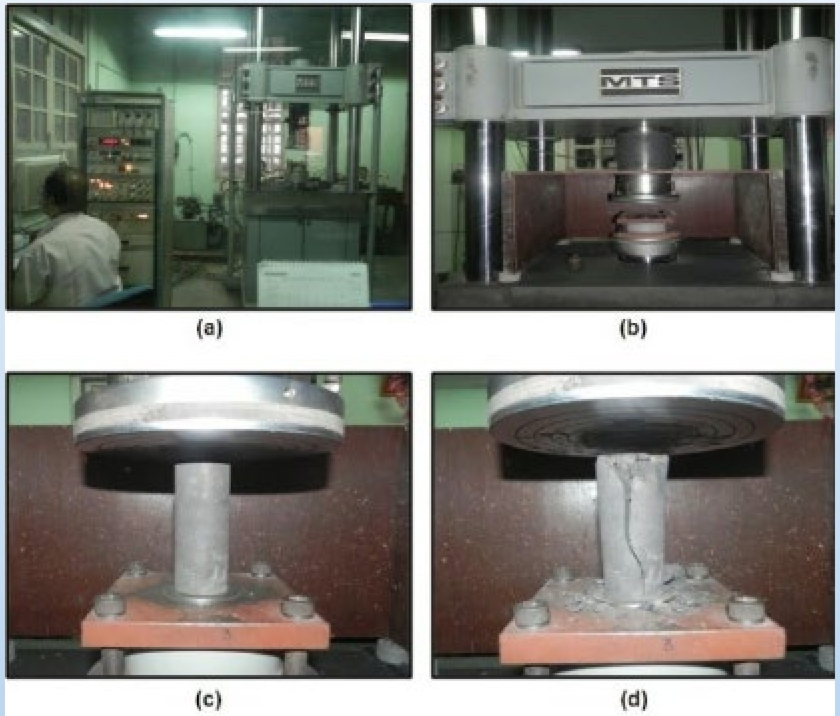
Cohesion – is it real?

- It depends. When unsure, neglect it in the drained, long-term case.
- What might seem like cohesion, but is not:
  - Soil suction from not being saturated.
  - Unloading of dense material.
  - Shearing of dense grains.
- What is actually cohesion:
  - Cemented grains.

When in doubt, $c'=0$!

Rock Shear Strength

- Conventionally, Hoek-Brown Model is used for assessing the shear strength of rock.
- Developed initially for tunneling applications, used for assessment of rockslope stability and foundations now.
- Big difference is the incorporation of tensile strength and fracturing/competence of geomaterials.
- Can be determined from uniaxial compressive testing of rock cores.



Hoek-Brown Failure Criteria

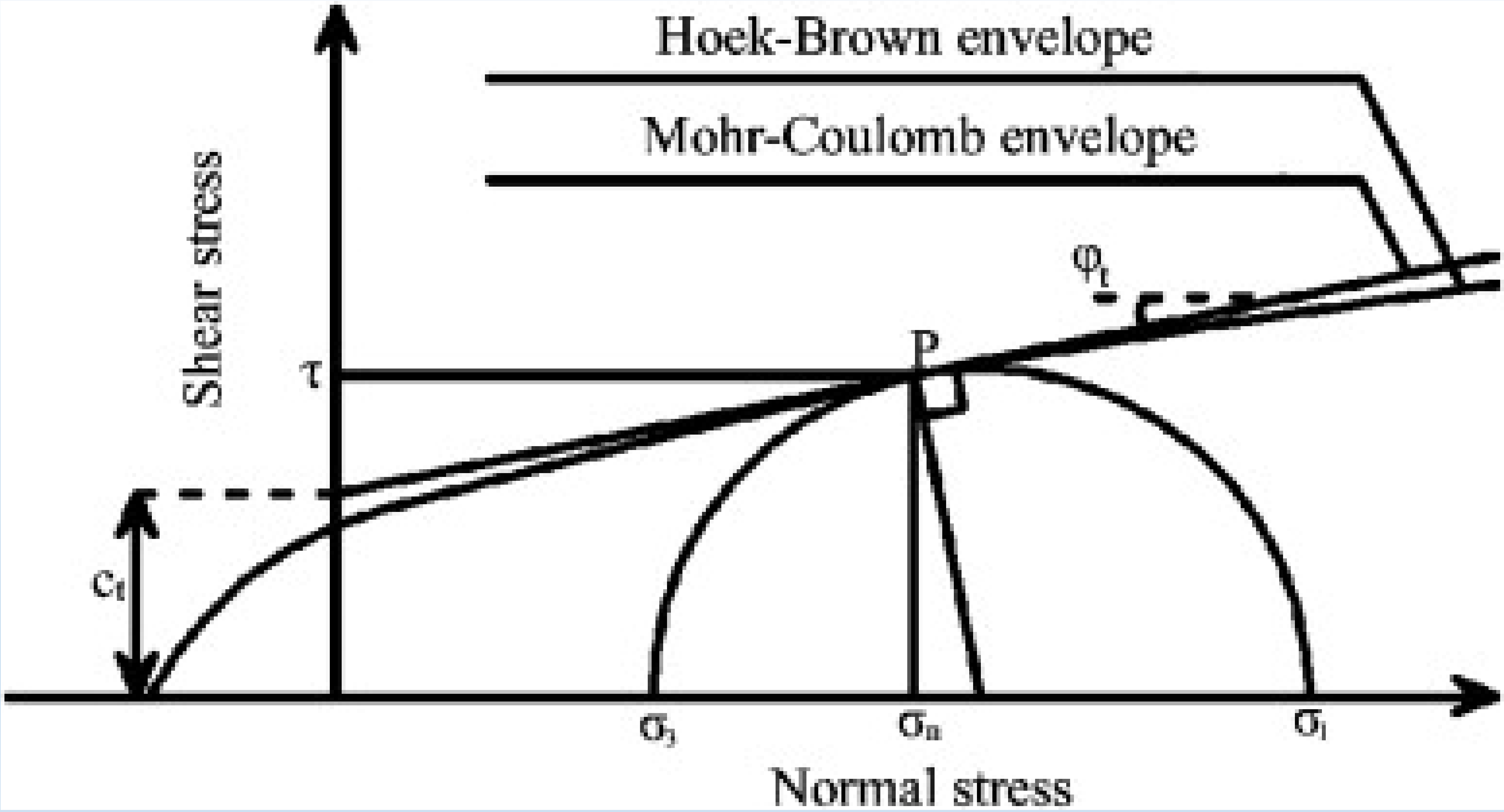
- Typical Hoek-Brown failure envelope defined as:

$$\sigma_1{}' = \sigma_3 + \sigma_{ci}{}' \left(m \frac{\sigma_3{}'}{\sigma_{ci}{}'} + s \right)^{0.5}$$

- $\sigma_{ci}{}'$ is uniaxial compressive strength of intact rock material.
- *m* and *s* are material constants, *s*=1, *m*=m_i for intact rock. For less competent rock:

$$m = m_i \exp\left(\frac{GSI-100}{28-14D}\right) \quad \text{and} \quad s = \exp\left(\frac{GSI-100}{9-3D}\right)$$

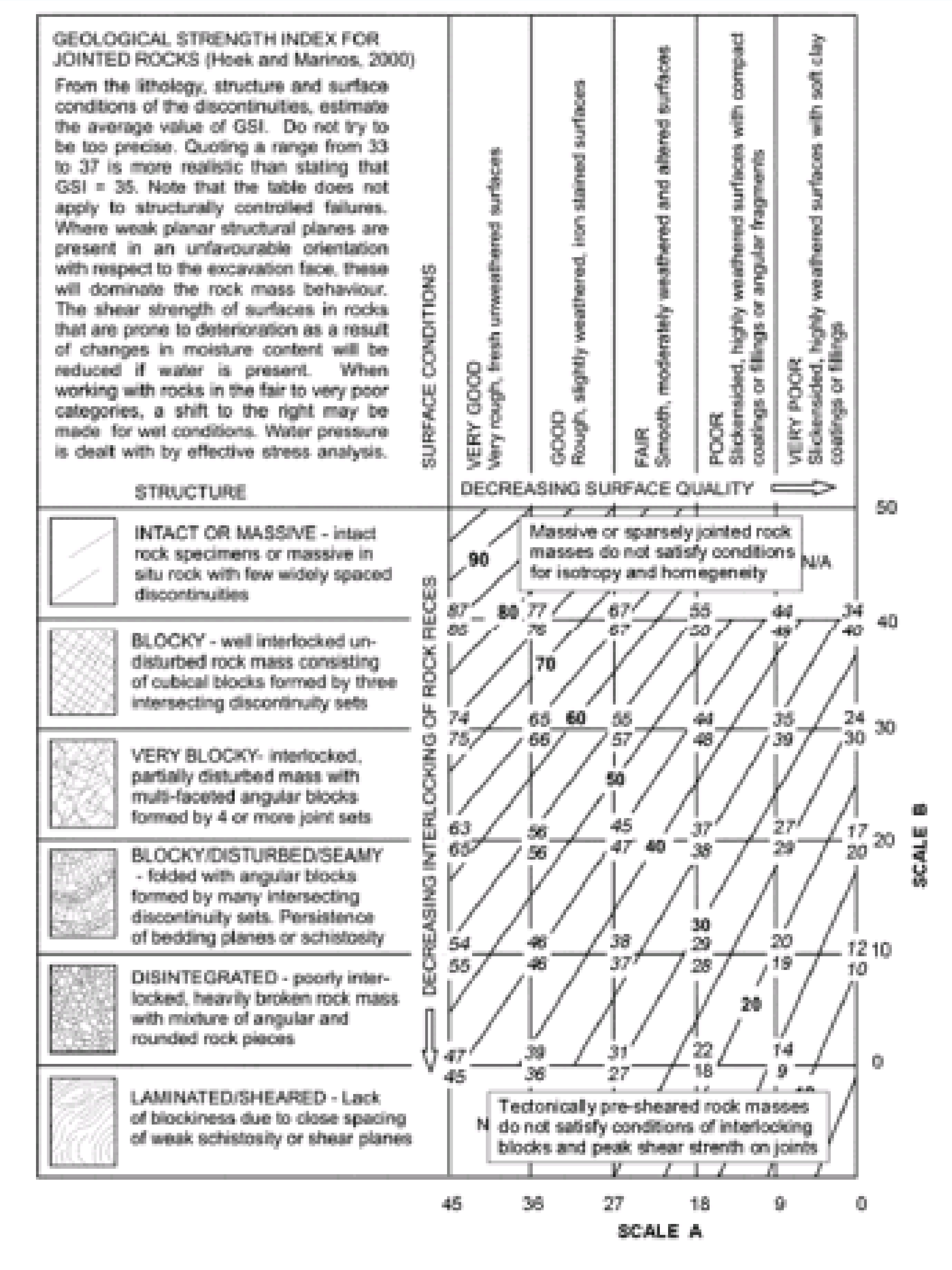
- *GSI* is Geological Strength Index, based on field interpretation.



Hoek-Brown envelope
Mohr-Coulomb envelope
Shear stress
τ
P
φt
ct
σ3
σn
σ1
Normal stress

Geological Strength Index

- Geological strength index, GSI, is a function of seams, fractures, laminations, weathering.
- Empirical, semi-quantitative.
- Rock is a poorly-characterized geomaterial.



Questions?

Application of Slope Stability Analysis - Theory

Standard Approach to Slope Stability

Learning Objectives

- Identify the process for which slope stability analysis is performed.
- Understand the influence of soil properties on slope stability.
- Understand the influence of slope geometry on slope stability.

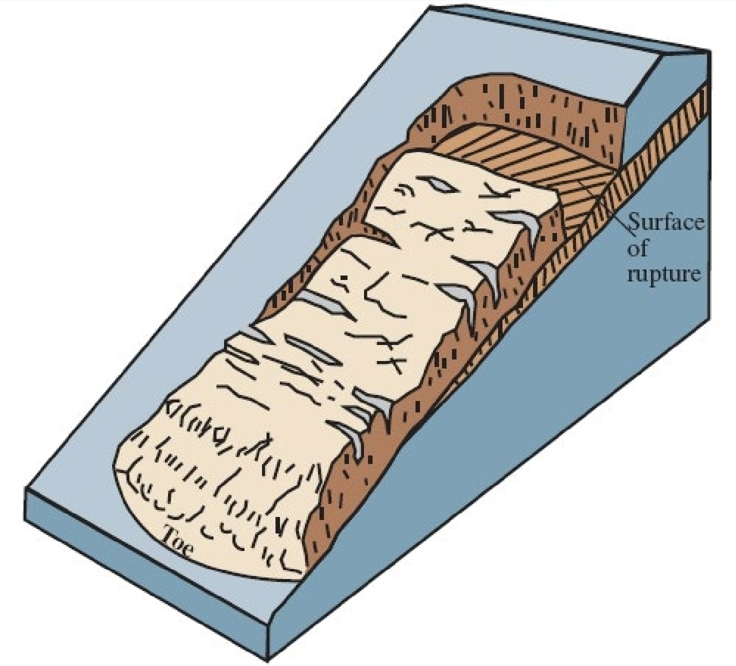
Concepts of Slope Stability

- Slope stability is the primary means of quantitatively assessing the level of stability of a slope, done using a Factor of Safety.
- Soil tends to fail in shear, these concepts directly govern slope failures.
- Soil has shear strength, conventionally defined as friction and cohesion.
- At a given shear surface, there is shear stress, induced by:
  - The gravitational mass of the soil.
  - Water pressures.
  - Overloading, seismicity, etc.
- Generally, slope stability is a comparison of available shear strength to shear stress:

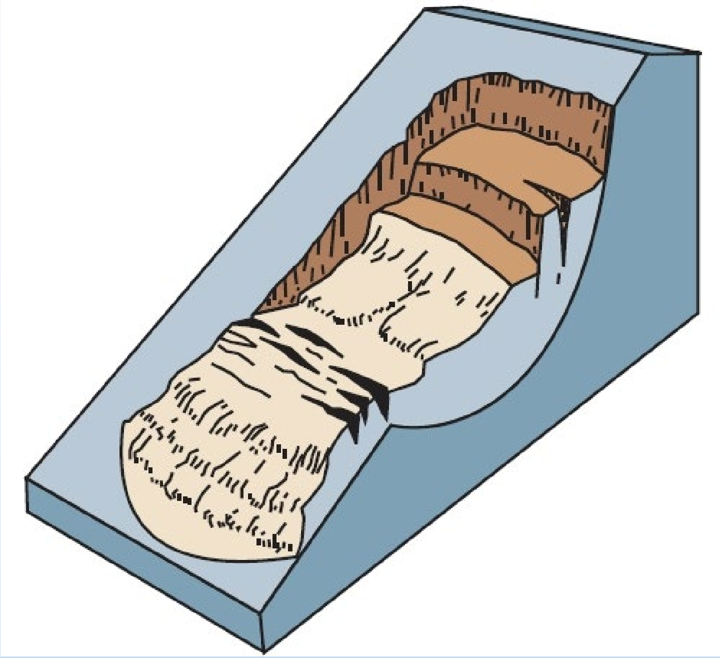
Factor of Safety = $FS = \frac{Available\ Shear\ Strength}{Mobilized\ Shear\ Stress}$

1. Identify the kinematics of the problem

- Kinematics – the study of the geometry of motion without regard for what caused it.
- For slope stability, the question is:
  - What is the likely initial geometry of motion that a potential slope failure would exhibit?
  - We may want to consider more than one possible failure.



Translational

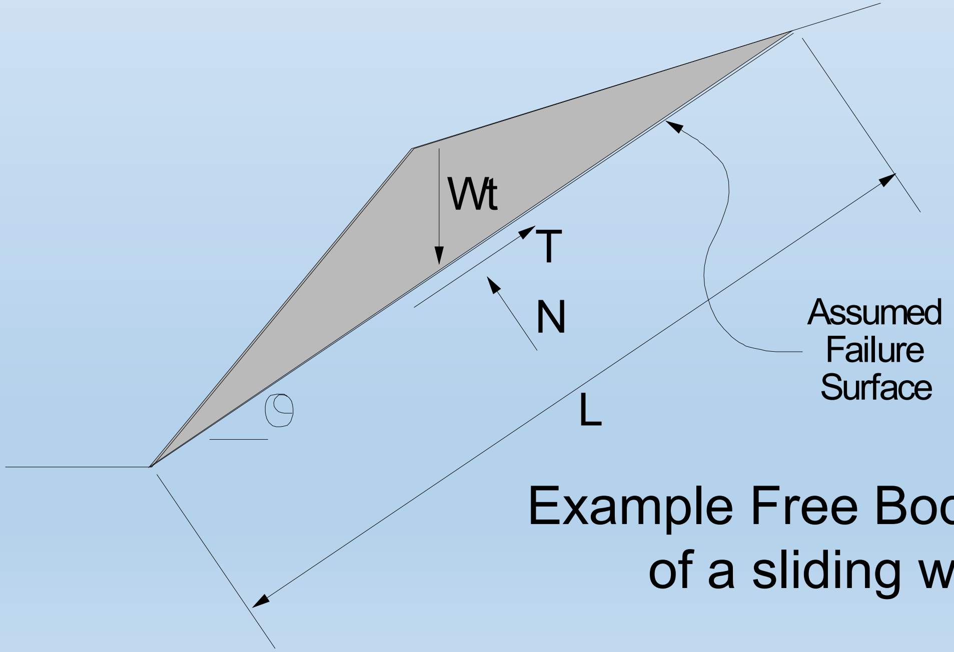


Rotational

Combination??

2. Construct a free body diagram of the rigid body of concern

- Isolate the Slide Mass.
- Indentify and show all external forces.
- Include body forces (weight and if an earthquake force is to be included, inertia).



Example Free Body Diagram of a sliding wedge.

3. Evaluate the forces on the free body diagram

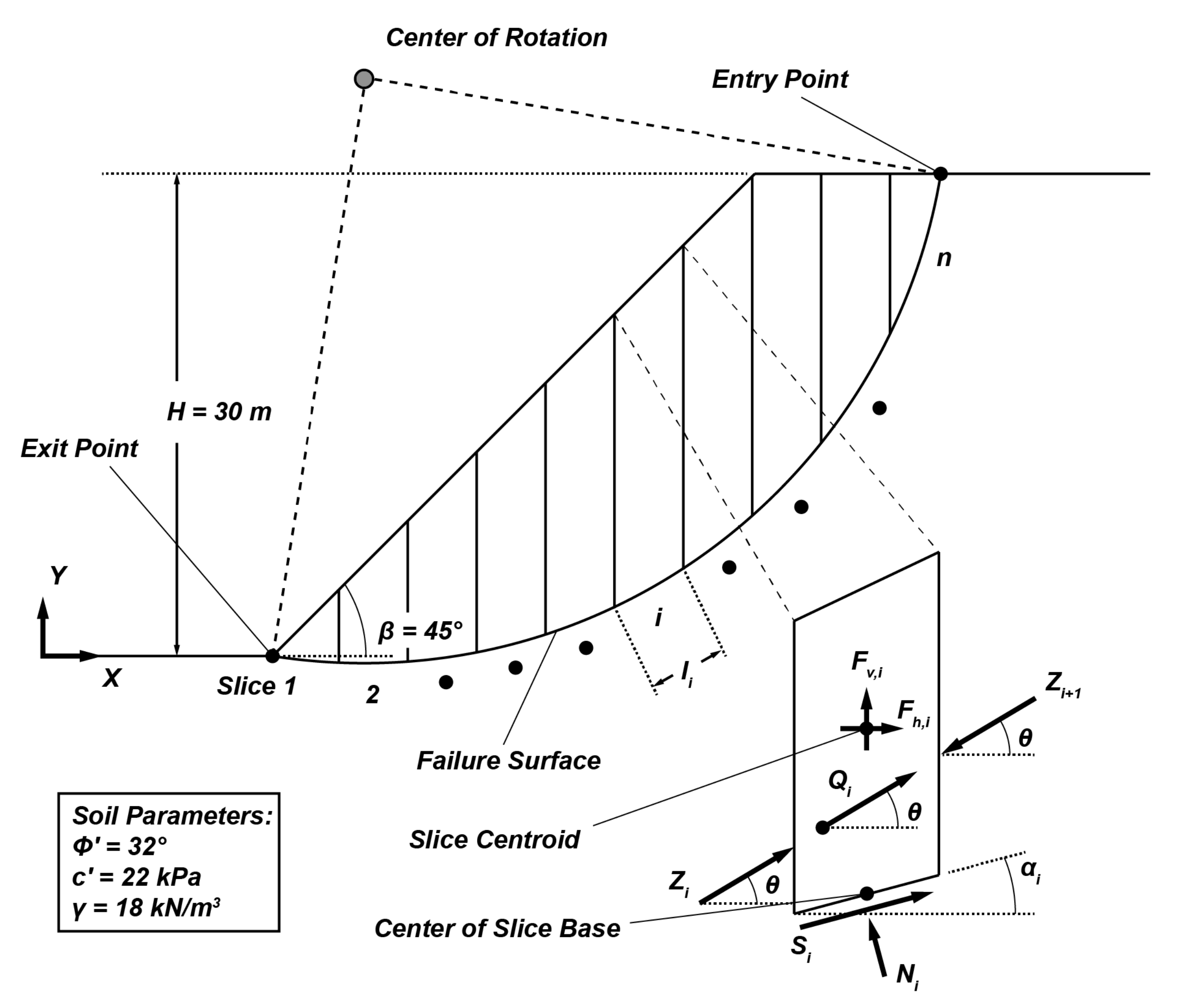
- ***Be Careful – some forces cannot be evaluated directly.***
- Weight of the body – in 2-dimensions, this will be the area times the appropriate unit weight.
- Boundary Neutral Forces – only present if a portion of the body is below the local GWT.
  - Compute the boundary pore pressure distribution.
  - Integrate the distribution to obtain a resultant force.
- Solve for other forces using equilibrium equations.
  - Usually requires simplifying assumptions.
  - Usually requires solution of simultaneous equations.

4. Incorporate a Factor of Safety to address cases where the slide mass is not in a state of limit equilibrium with soil strength fully mobilized.

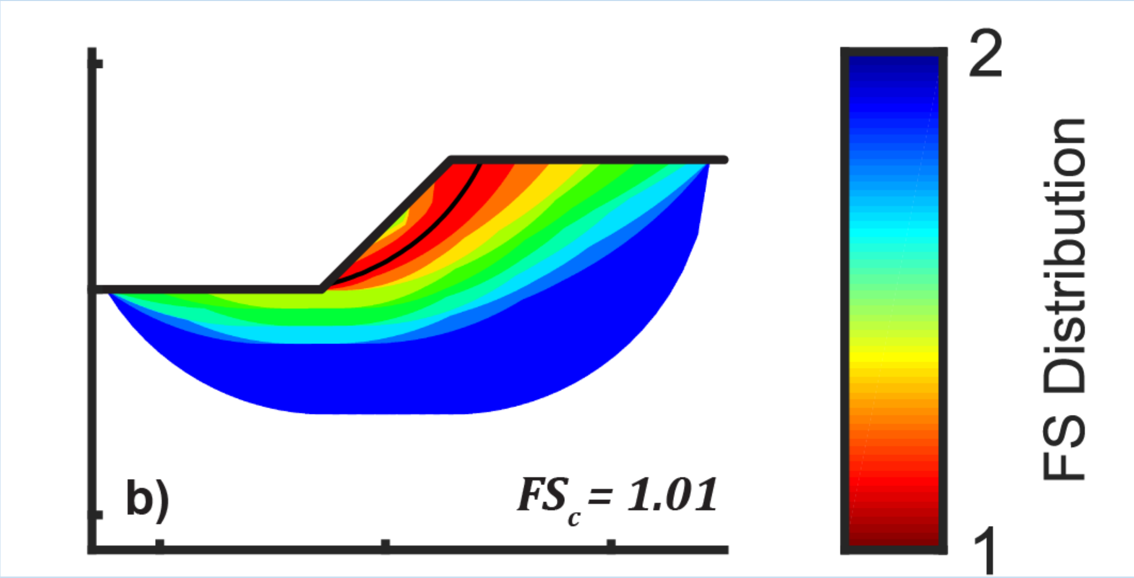
$$FS = \frac{Available\ Shear\ Strength}{Mobilized\ Shear\ Stress}$$

$$FS = \frac{Forces\ Resisting}{Forces\ Driving}$$

$$FS = \frac{Moments\ Resisting}{Moments\ Driving}$$



5. Analyze other similar free bodies in order to determine the worst case situation.
   - If failure surface is absolutely known this will not be required.
   - Failure surface is rarely “absolutely known”.
   - The exception is for existing landslides (forensics).
   - **To find the failure geometry of an existing landslide, we need to perform a field investigation and monitoring.**



SLOPE STABILITY ANALYSIS METHODS

SLOPE STABILITY ANALYSIS METHODS

1. Use stability charts for simple slopes analysis
2. List different limit equilibrium (LE) methods
3. Match common LE methods to appropriate failure modes

List of Typical LE Methods

- Infinite Slope
- Culmann Method (Planar Surface Analysis)
- Ordinary Method of Slice (Fellenius Method)
- Bishop Method
- Janbu Rigorous or Simplified Method
- Morgenstern-Price Method
- Spencer Method
- Corps of Engineers Method or Wedge-Method
- More…

Classified LE Methods by Level of Complexity

1. Simple
2. Simplified
3. Rigorous

Simple LE Methods

- Satisfy equilibrium
- Limited to homogeneous slopes
- Restrictive slip surface geometry
- Example: Infinite slope, Planar Surface Analysis (Coulomb), Log Spiral

Simplified LE Methods

- Do not satisfy equilibrium
- Can deal with layered soil
- In most cases restrictive slip surface geometry
- Example: Bishop, Ordinary Method of Slices, Friction Circle Method (ϕ-Circle), Multiple Wedge

Rigorous LE Methods

- Satisfy equilibrium
- Effects of statical assumptions can be assessed
- Can deal with layered soil
- General shape slip surface geometry
- Example: Morgenstern-Price, Janbu, Spencer

Why are there so many methods?

- LE is statically determinate only for extremely simple problems

- Geometry of potential slip surfaces can be complex

Review of **Simple** LE Methods

1. ***Infinite Slope***
2. ***Planar Surface Analysis (Culmann)***
3. ***Sliding Block***
4. ***Log Spiral***

Infinite Slope Stability Analysis

- Simple problems facilitate and reinforce the understanding of ‘abstract’ concepts
- Infinite slope problems are simple to formulate
- The results are instructive and could be useful in certain cases

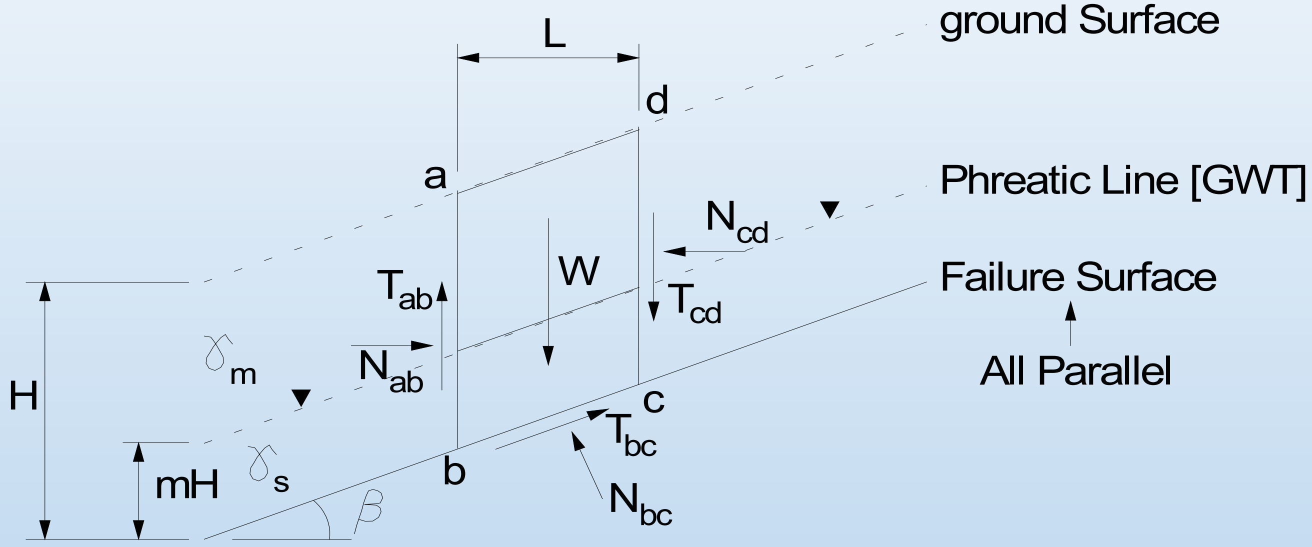
Development of the Infinite slope equation

Free Body Diagram

γ_m = moist unit weight

γ_s = saturated unit weight

m = decimal fraction



Assumptions:

- A slice from the slope [shown] is representative of the whole slope.
- Side forces are equal and opposite, and will therefore cancel in any force equilibrium equations.
- Two dimensional analysis ignores forces acting on the plane of the section.

The resulting Factor of Safety Equation for the Infinite Slope Case is:

$$FS = \frac{c'}{H((1-m)\gamma_m + m\gamma_s)\sin\beta\cos\beta} + \left[\frac{(1-m)\gamma_m + \gamma'_s\, m}{(1-m)\gamma_m + \gamma_s m}\right]\frac{\tan\phi'}{\tan\beta}$$

The equation will work for any GWT level as long as the ground surface, GWT, and failure surface are parallel.

If the soil is cohesionless [c’ = 0] then the Factor of Safety equation is actually dimensionless.

Let's examine the relationship for a special case:

Consider a cohesionless soil, c'=0 (a sand or silt sand for example)

The Factor of Safety equation reduces to:

$$FS = \left[\frac{(1-m)\gamma_m + \gamma'_s\, m}{(1-m)\gamma_m + \gamma_s m} \right] \frac{\tan\phi'}{\tan\beta}$$

Now consider that: $\gamma_s \approx 2\ times\ \gamma_w$

hence: $\gamma'_s \approx \frac{1}{2}\gamma_s$

For m = 0 [no water]

$$FS = \frac{\tan\phi'}{\tan\beta}$$

For m = 1 [seepage to the ground surface]

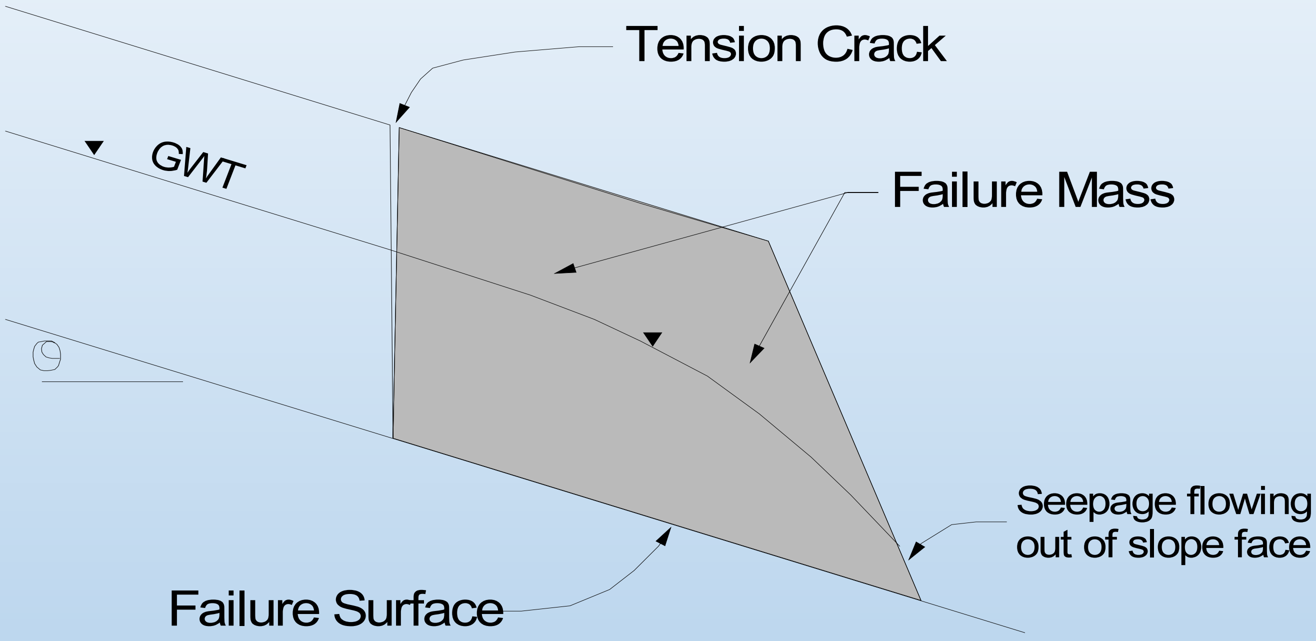
$$FS = \frac{1}{2} \cdot \frac{\tan\phi'}{\tan\beta}$$

Sliding block failure – including the effect of water.

- Applicable in only selected field cases where there is a definable failure surface.
- Very similar to the wedge in formulation
- Allows us to explore the influence of water through the use of boundary neutral forces
- Allows us to explore the influence of cohesion.

Forces us to consider the mechanism of tension cracks and their influence.

Sliding Block



What is a tension crack?

Tension cracks come in a range of sizes.

Small

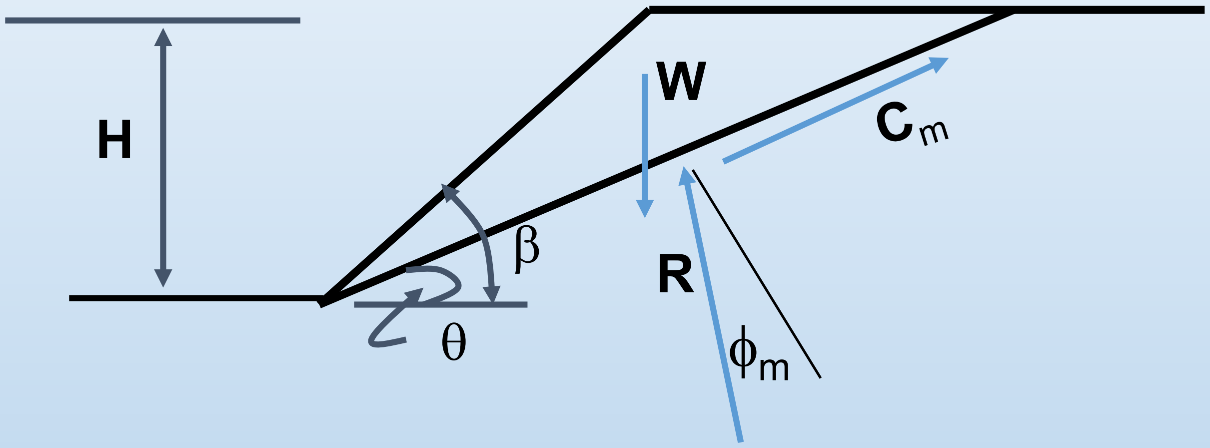
Hard to see.

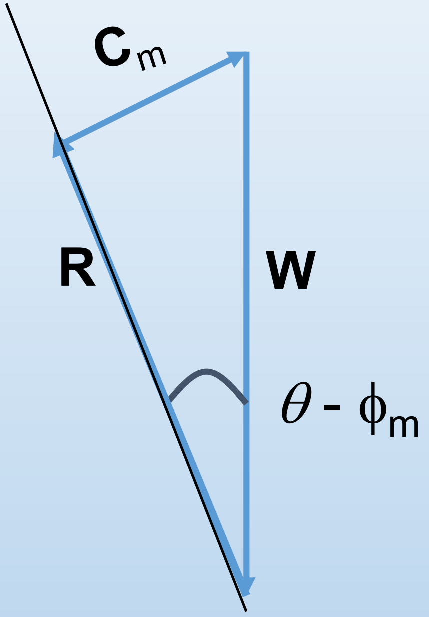
Large

A tension crack will form in an extension zone in a soil that possesses cohesion when the mass is near failure

- Usually we must assume the depth
  - Common assumption $$D_{crack} = \frac{2c'}{\gamma_{soil}}$$
  - Maximum assumption $$D_{crack} = \frac{4c'}{\gamma_{soil}}$$
- A tension crack will fill with water to the level of the local GWT, and possibly to the ground surface if surface water is allowed to flow into it.

Finite Height Slope: Planar Wedge Failure Analysis (Culmann 1866)

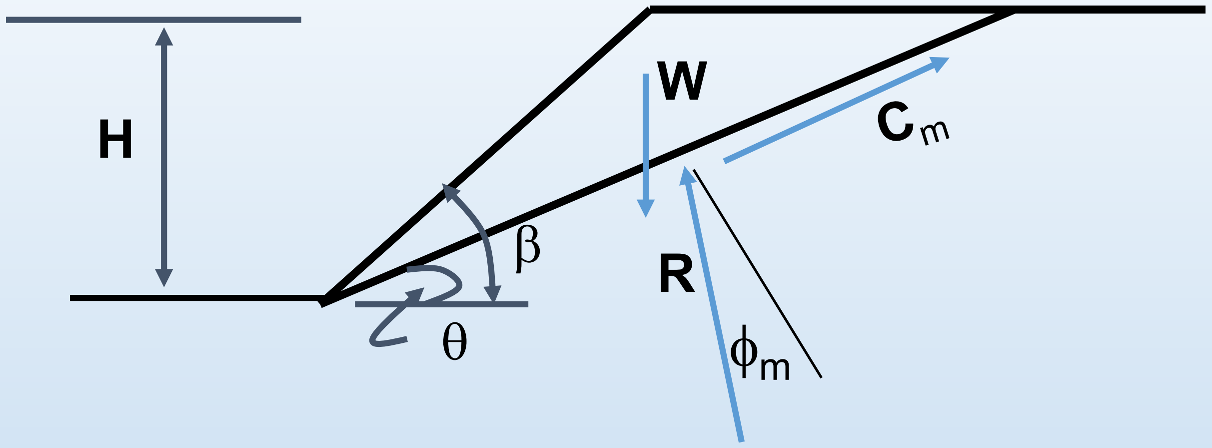




Assumptions:

- Planar Failure Surface.
- Two dimensional analysis considers only the forces shown on a unit slice in the third dimension.
- Water is usually not included in the formulation.

For an assumed FS, draw the force polygon



$$FS = \frac{c' L + W \cos\theta \tan\phi'}{W \sin\theta}$$

Use this expression while varying θ to find the minimum Factor of Safety and the failure surface that corresponds to it.

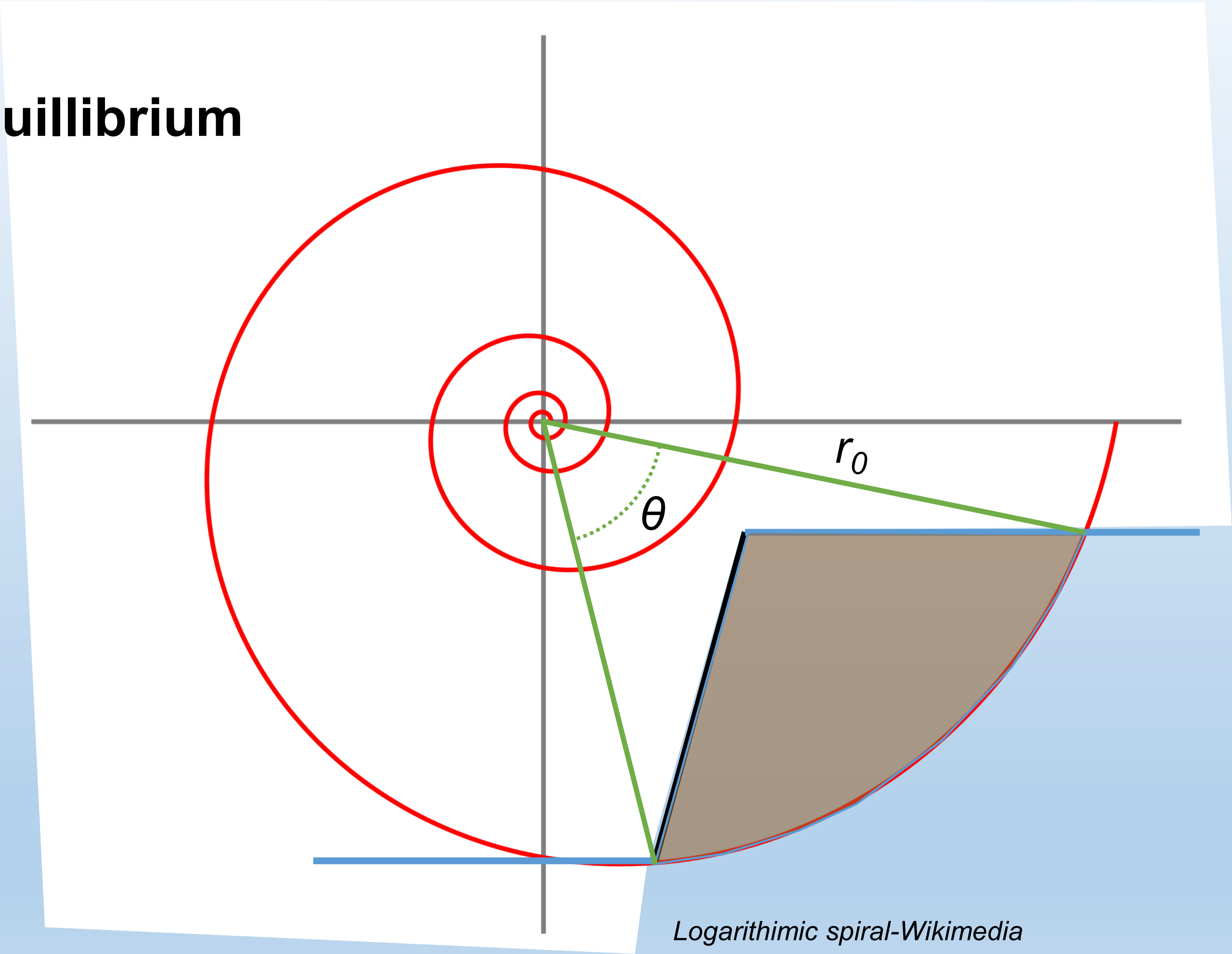
This is the Factor of Safety for the slope.

Log-Spiral Limit Equillibrium

$$r = r_0 e^{\theta\, tan(\phi_d)}$$

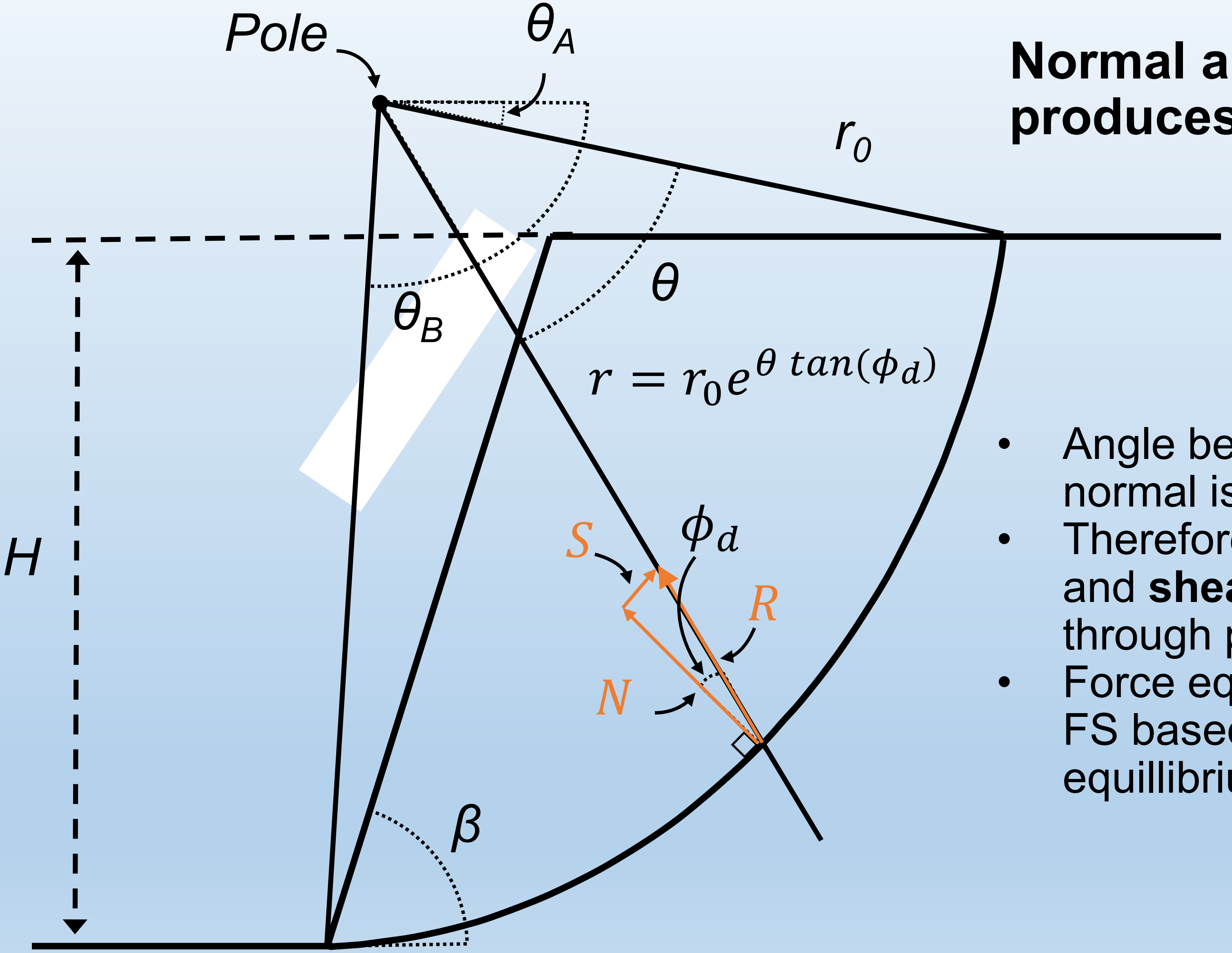
The constant a is replace by r_0 and the constant b is replaced by ϕ_d

Using a segment of the log-spiral perform single free-body limit equillibrium



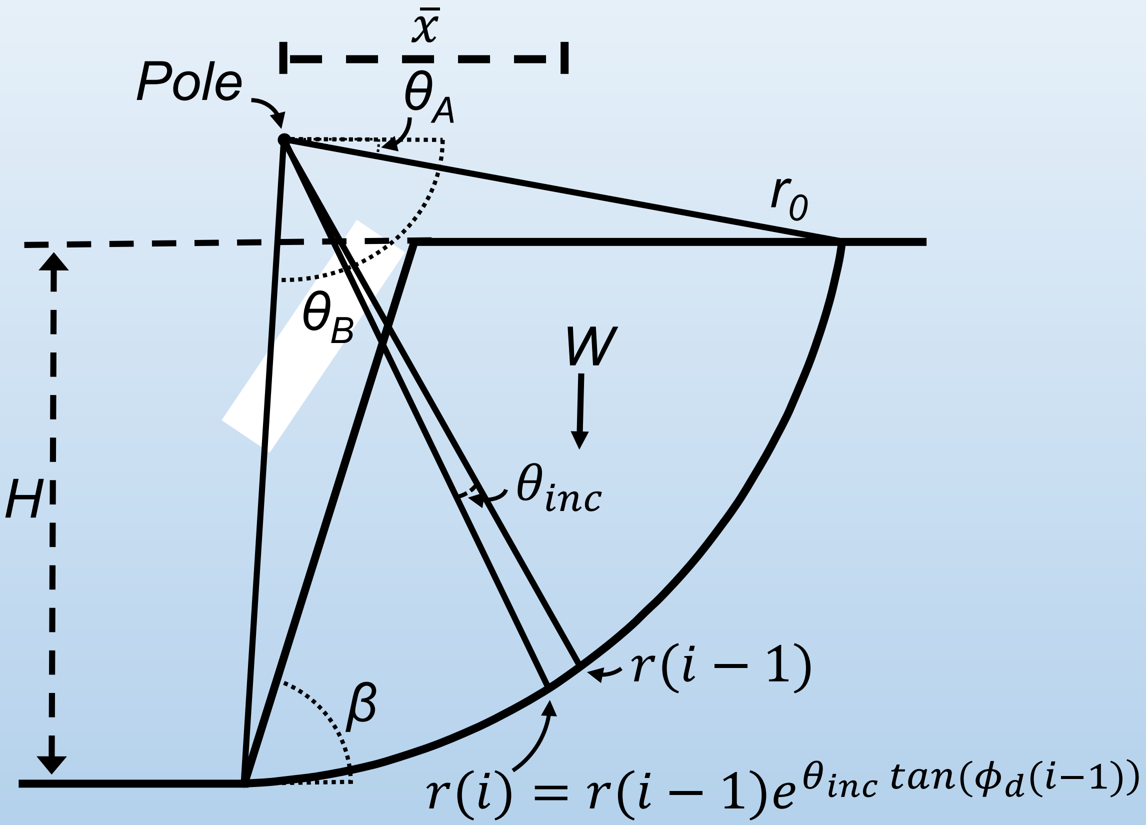
Logarithimic spiral-Wikimedia

Normal and shear forces produces net zero moment



- Angle between radius and normal is always ϕ_d
- Therefore, **resultant** of **normal** and **shear** force always passes through pole
- Force equilibrium satisified so FS based on moment equillibrium

Strength Reduction FS Determination



- Resisting Moment (M_c)

$$M_C = \sum_{i=1}^{n} c_{d:i} \Delta l_i r_i \cos(\phi_{d:i})$$

- Driving Moment (M_w)

$$M_W = W \cdot \bar{x}$$

- Use definition of mobalized cohesion and friction angle

$$c_d = \frac{c}{FS} \qquad \tan(\phi_d) = \frac{\tan(\phi)}{FS}$$

- Iterate FS until Mc/Mw=1.0

Summary of Simple LE Methods

Method	*Failure Surface*	*Soil*	*Satisfy LE*	*Comments*
Infinite	*Planar*	*Uniform*	*Yes*	*Reasonable for surficial stability*
Culmann/ Wedge	*Planar*	*Uniform*	*Yes*	*May be under-conservative for* $\beta<75°$
Log Spiral	*Log Spiral*	*Uniform*	*Yes*	*Slip surface resembles circular arc*

Overview of Simplified LE Methods

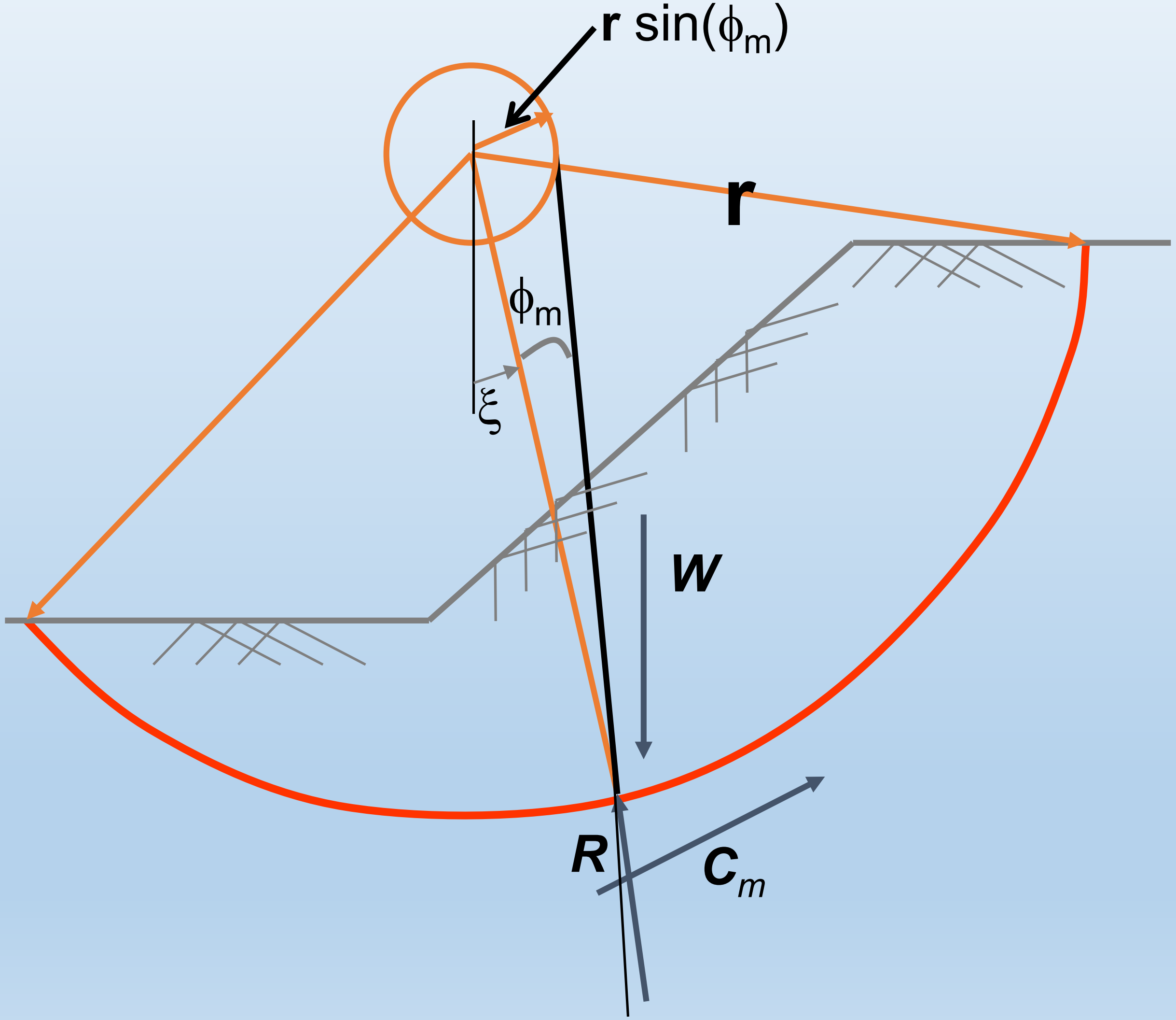
1. *ϕ-Circle and Taylor Stability Chart*

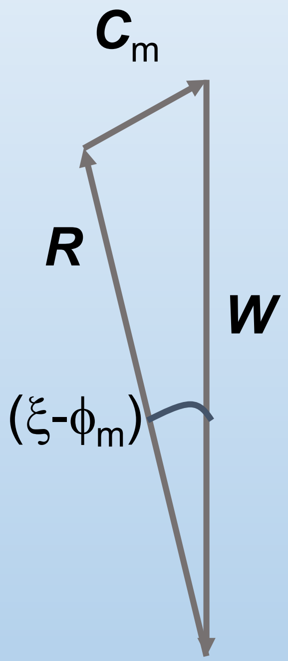
2. Ordinary

3. *Bishop*

4. Simplified Janbu

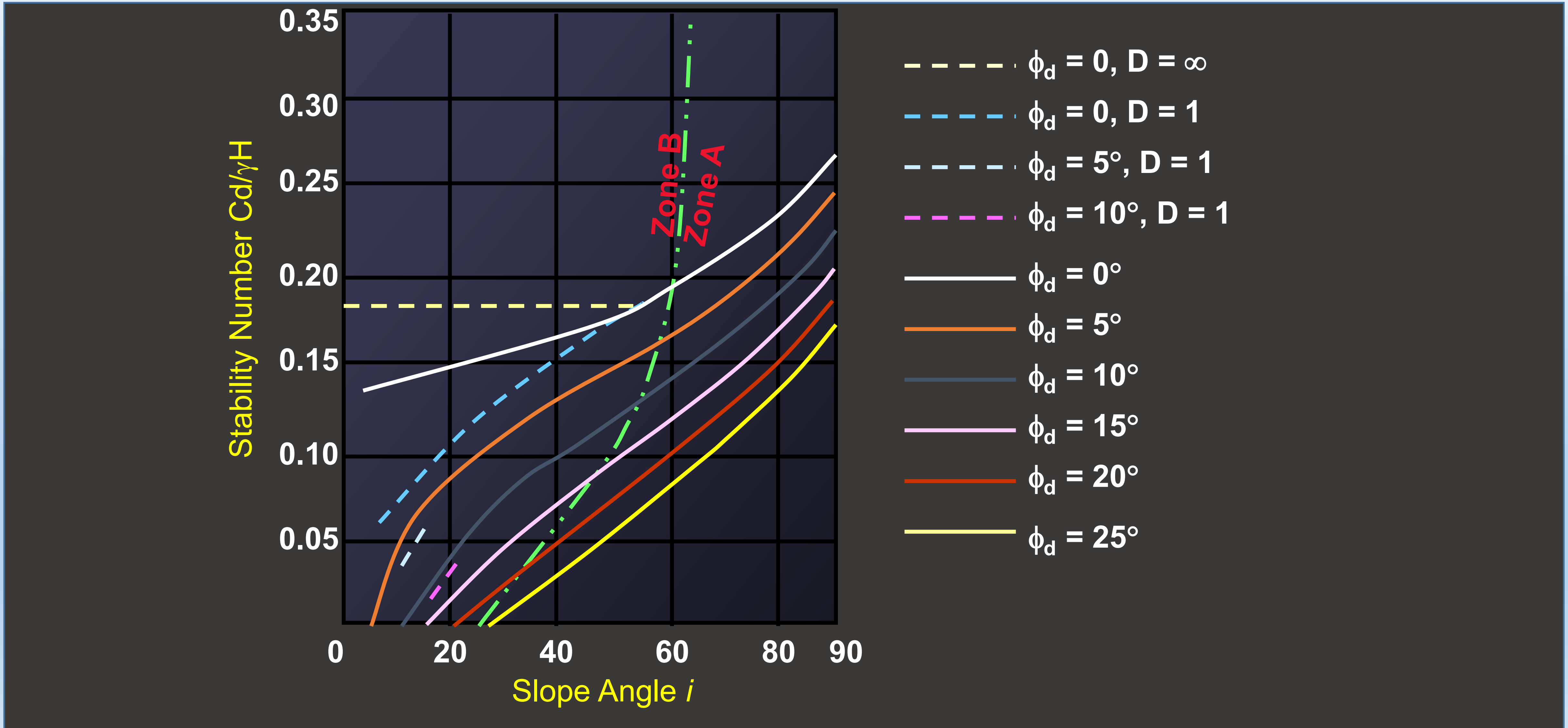
"ϕ - Circle" Method (Taylor, 1937)





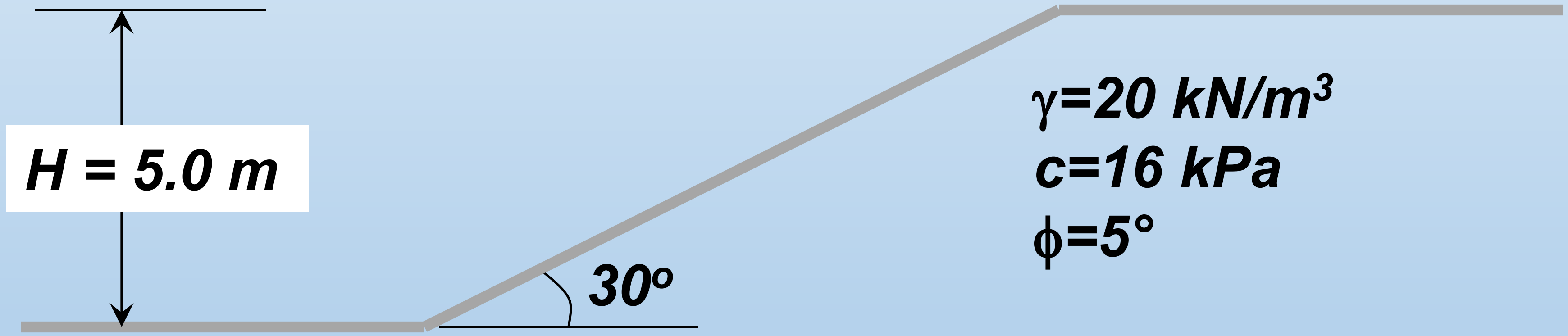
Taylor Design Chart

$$c_m = c_d = c / FS \quad \phi_m = \phi_d = \tan^{-1}[\tan(\phi) / FS]$$



Example: Use of Taylor Chart

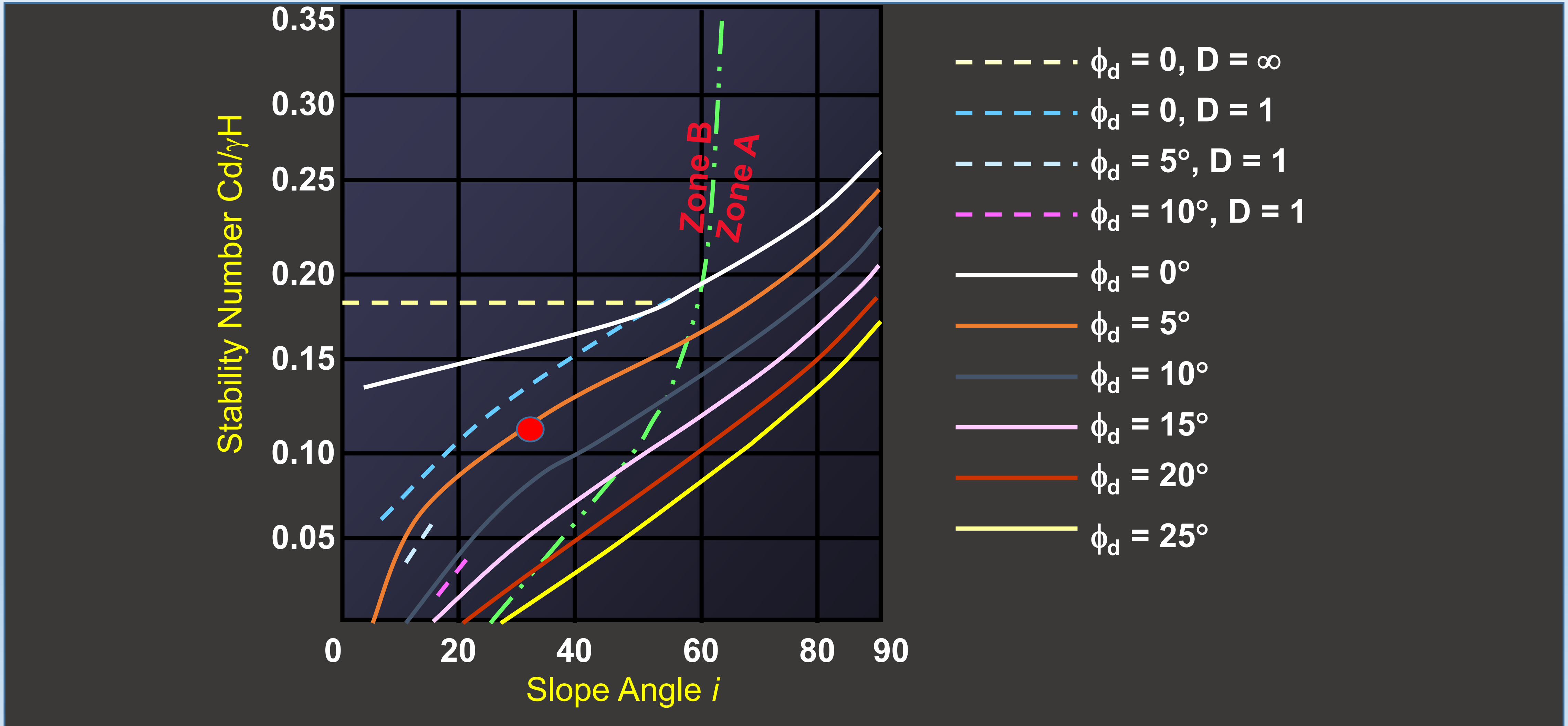
Determine FS for the given slope:



- Assume FS =1.0 → $\phi_d = \tan^{-1}(\phi) = 5^o$ →

Chart $(c_d/\gamma H) = (c/FS\gamma H) = 0.11$ → $FS = c/0.11\gamma H$

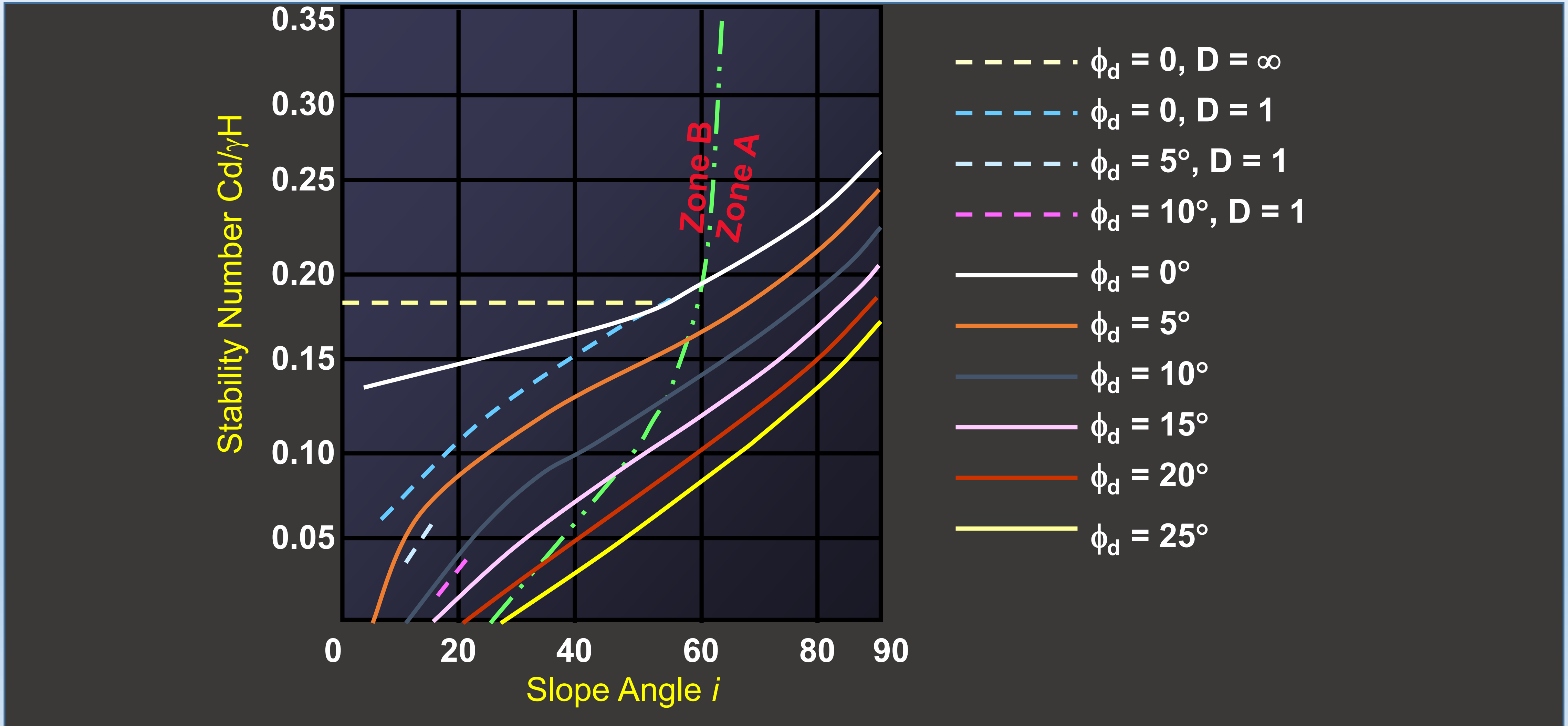
Resulted FS = 1.45 → Assumed FS≠ Resulted FS



Assume FS =1.2 → $\phi_d = \tan^{-1}[\tan(\phi)/FS] = 4.1^o$ →

Chart $(c_d/\gamma H) = (c/FS\gamma H) = 0.12$ →

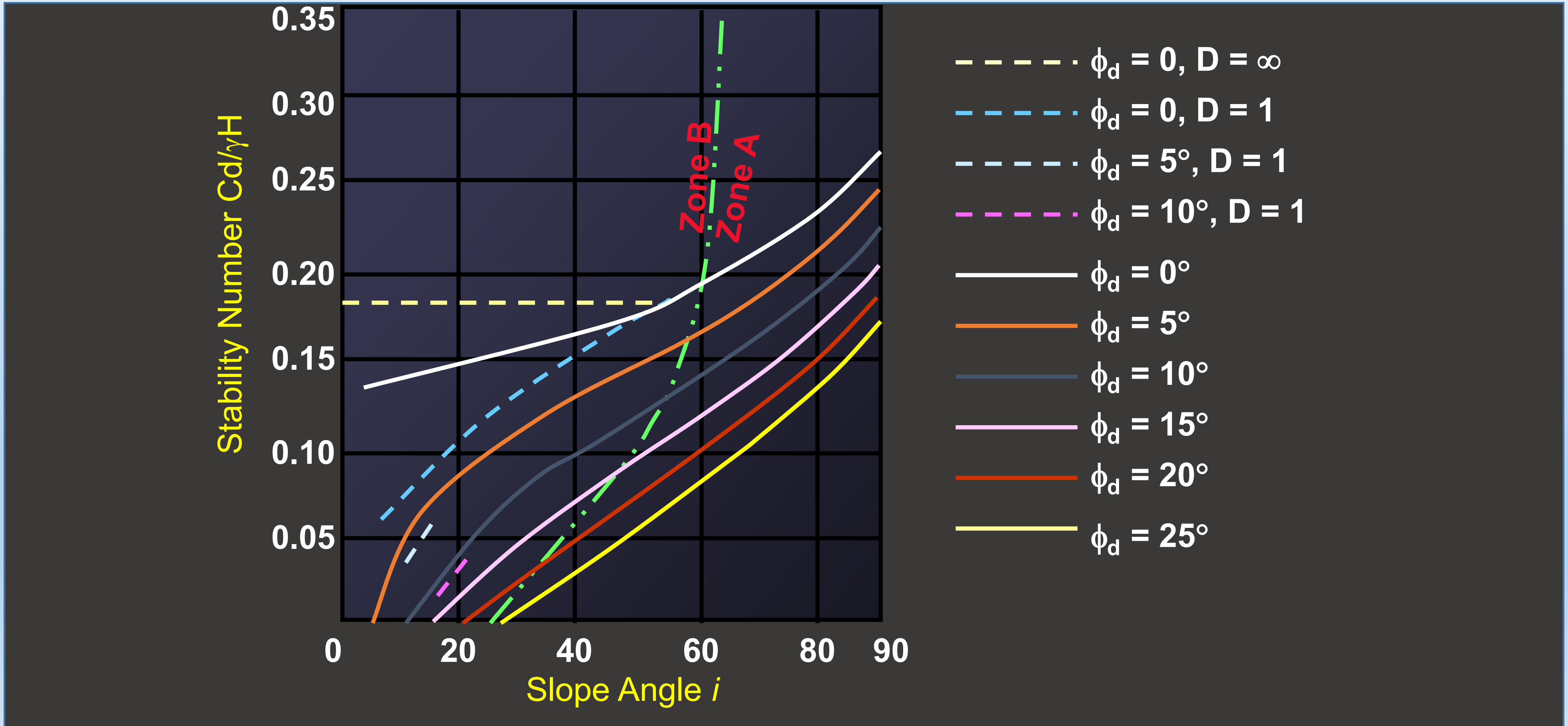
Resulted FS = 1.33 →Assumed FS≠ Resulted FS



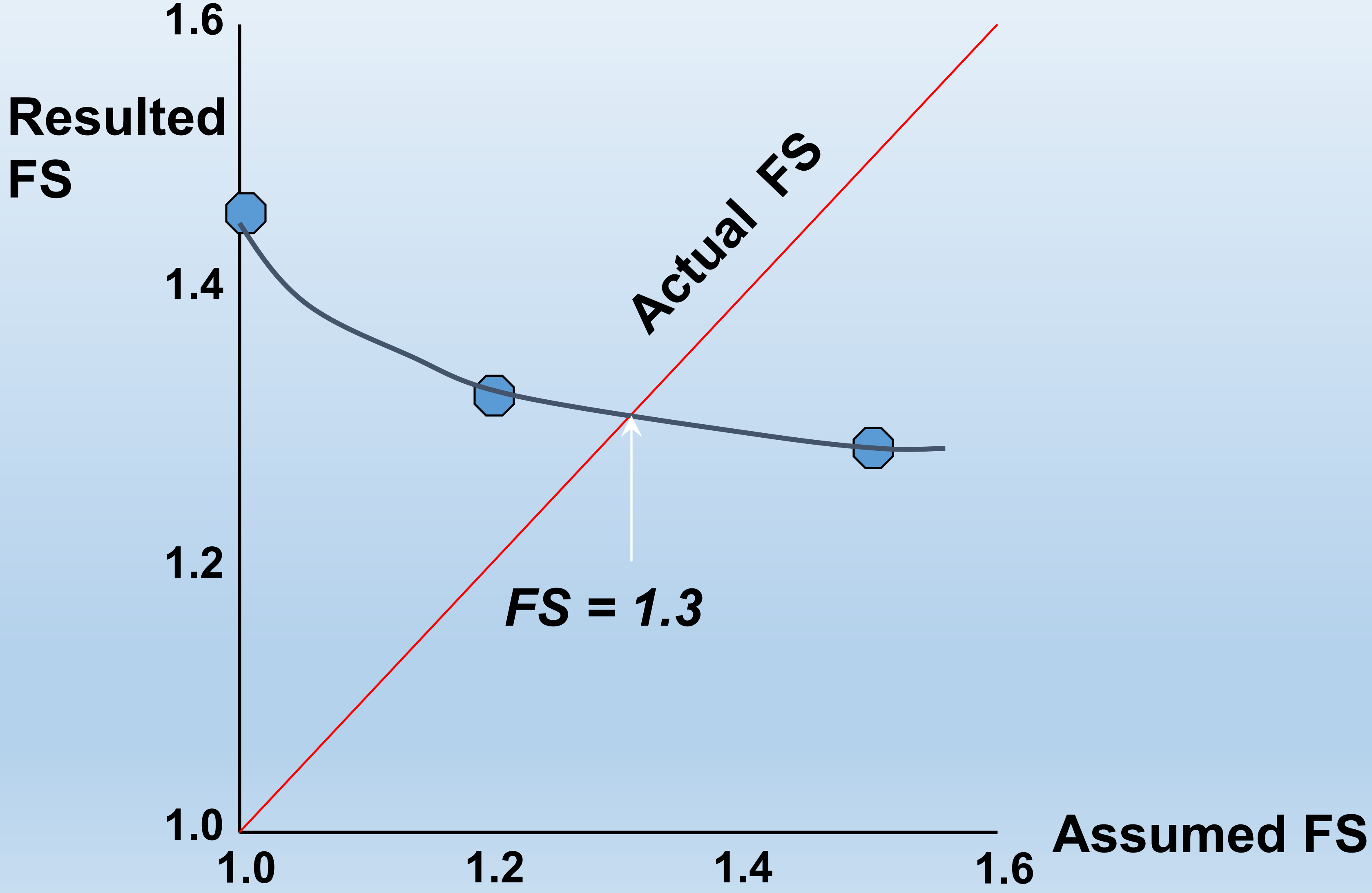
$FS = 1.5 \rightarrow \phi_d = \tan^{-1}[\tan(\phi)/ FS] = 3.3^o$

$\rightarrow$ Chart $(c_d/\gamma H) = (c/FS\ \gamma H) = 0.126 \rightarrow$

$FOS = 1.28 \rightarrow$ Assumed $FS \neq$ Resulted FS

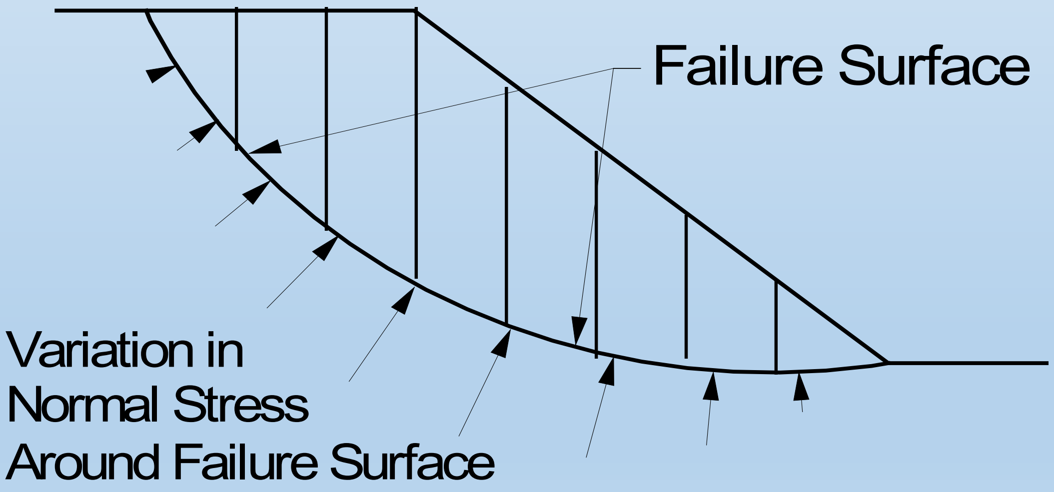


Example: Solution

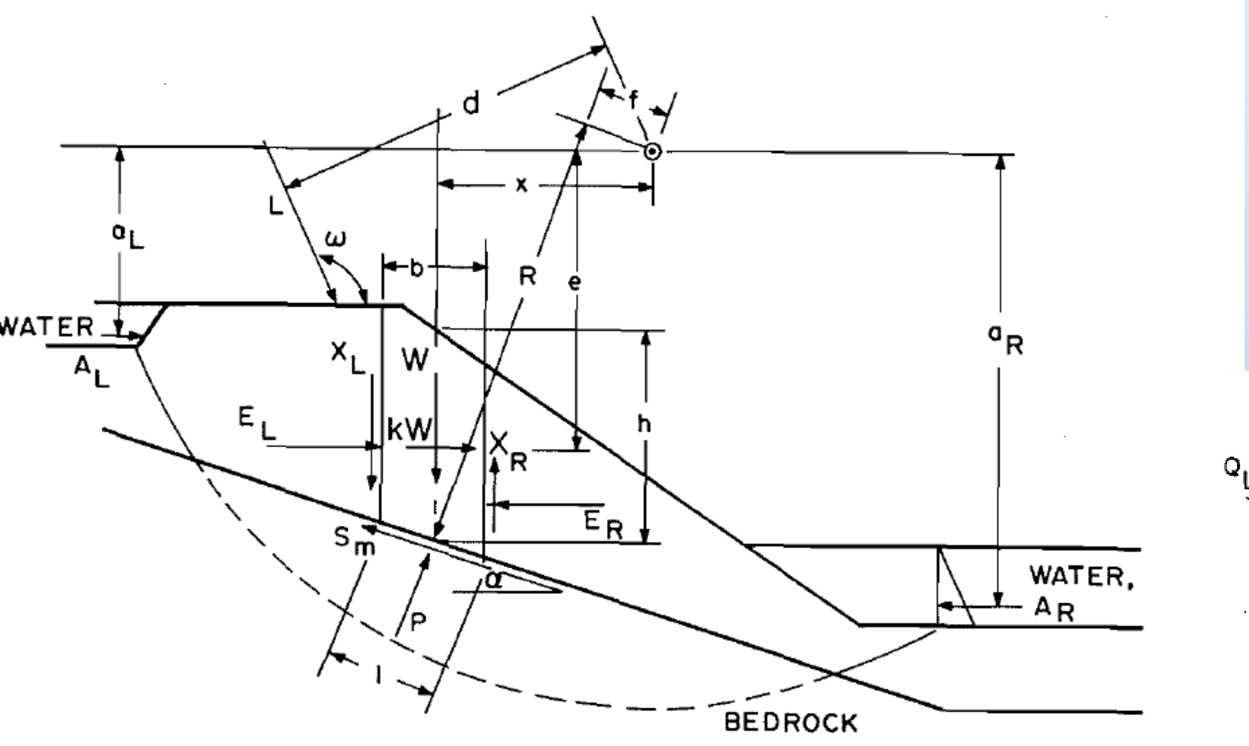


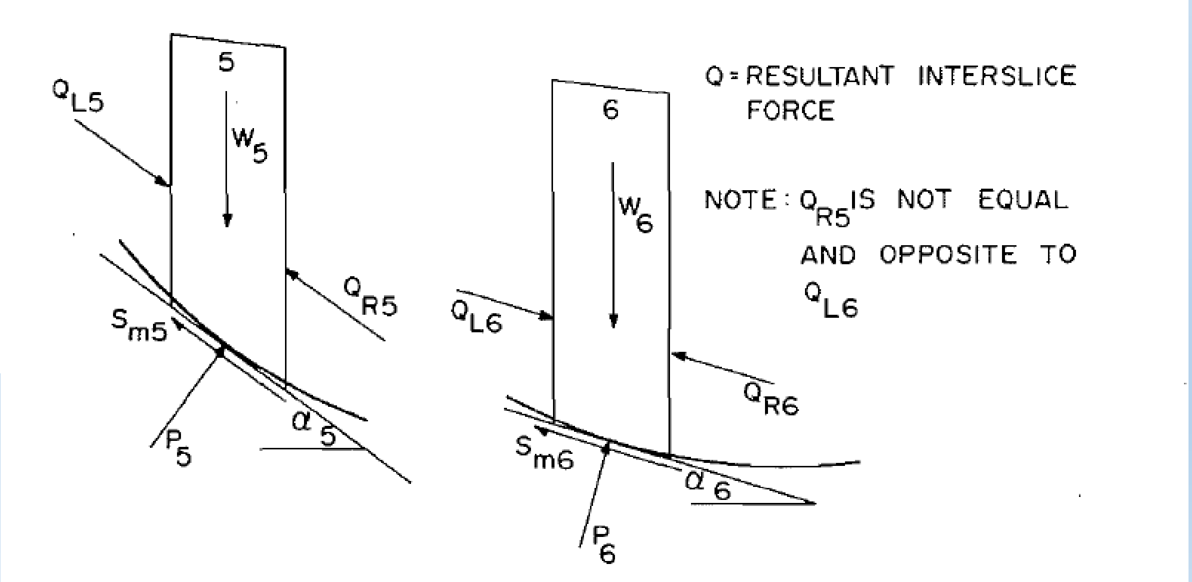
Method of Slices

- Effective normal stresses vary around the failure surface.
- It is assumed that a reasonable representation of effective normal stress around the failure surface can be obtained by dividing the slide mass into slices.



Ordinary Method of Slices (Fellenius 1927)





Ordinary Method of Slices (Fellenius 1927)

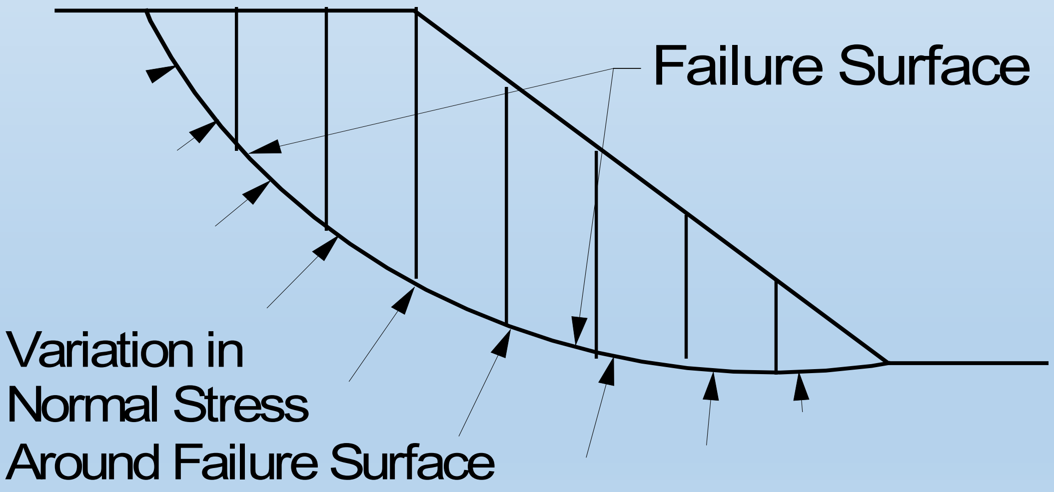
- Limited to circular arcs only
- Based on sum of moments around a point of rotation
- Does NOT satisfy force equilibrium
- Simple to Apply

$$FS = \frac{\sum\left[\frac{c' \cdot b}{\cos\alpha} + (W\cos\alpha - u \cdot b \cdot \cos\alpha)\tan\phi'\right]}{\sum W\sin\alpha}$$

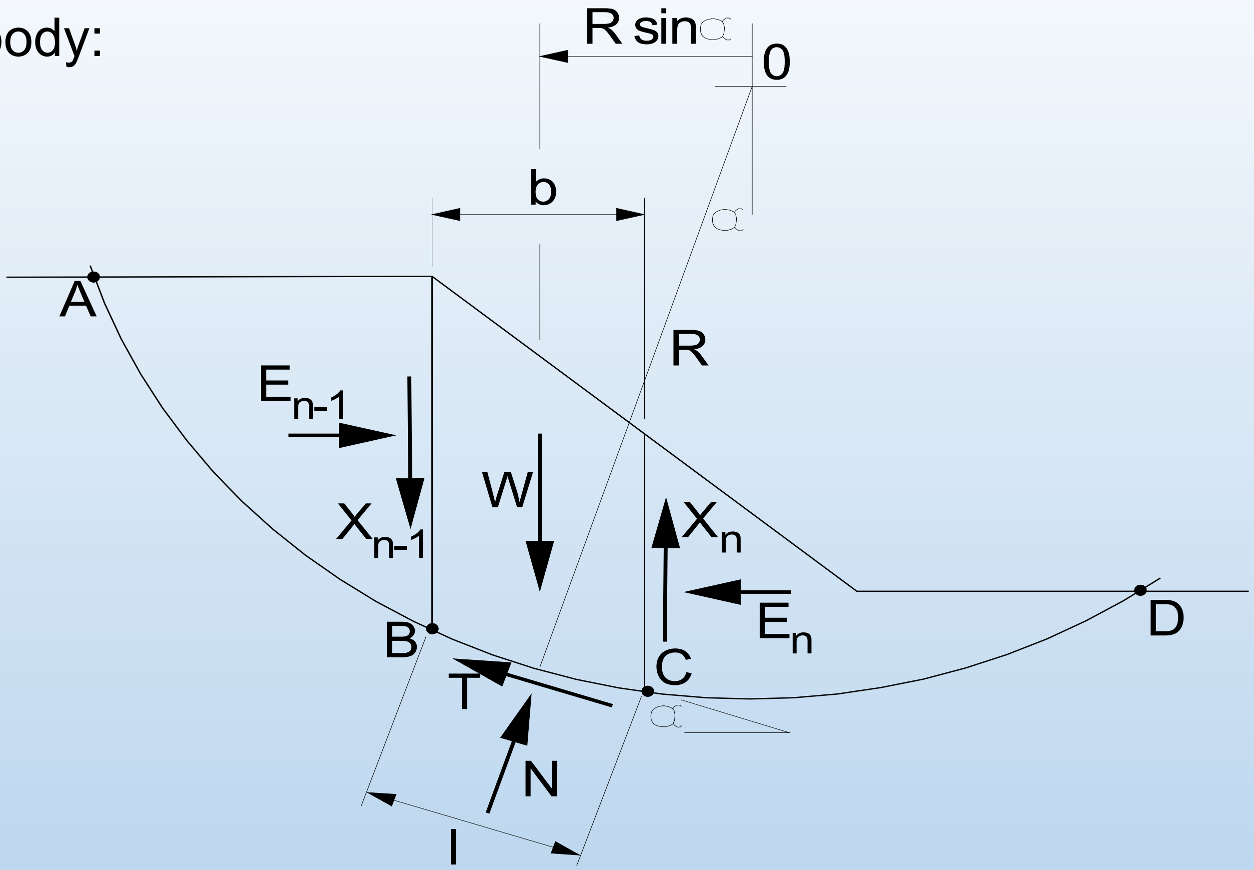
Bishop Simplified Method

- Limited to circular arcs only
- Satisfies moment and vertical force equilibrium
- Neglects shear forces in the vertical direction within the sliding mass
- Most popular method. Generally, it yields close results to rigorous analysis!

- Bishop's Method is a "method of slices".
  - Effective normal stresses vary around the failure surface.
  - It is assumed that a reasonable representation of effective normal stress around the failure surface can be obtained by dividing the slide mass into slices.



Slice Freebody:



Sum Moments of the external and body forces that act on the entire slide mass about the circle center.

$$\sum W \cdot R \cdot \sin\alpha - \sum T \cdot R = 0$$

The Factor of Safety from what is commonly referred to as Bishop's Method:

$$FS = \frac{\sum \left[\dfrac{c'b + (W - ub)\tan\phi'}{\cos\alpha \left(1 + \dfrac{\tan\alpha \tan\phi'}{FS} \right)} \right]}{\sum W \sin\alpha}$$

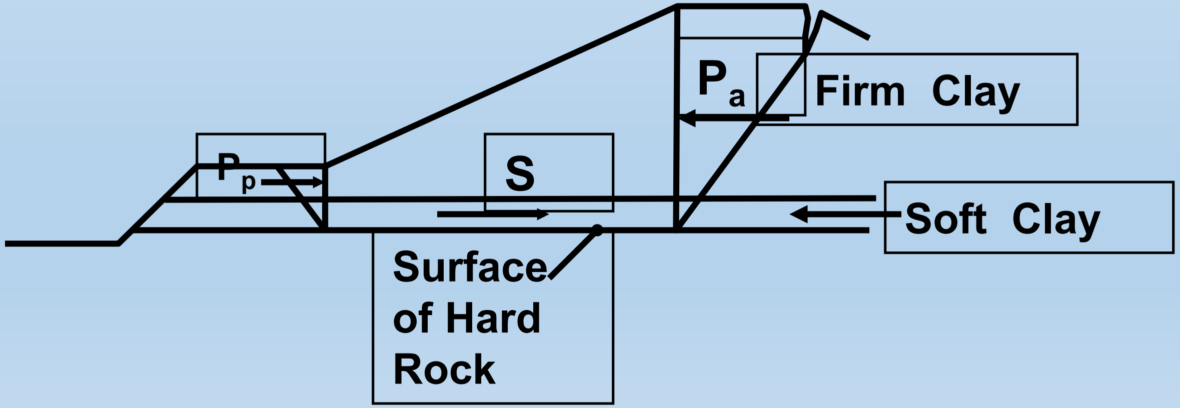
You will note that the Factor of Safety is on both sides of the equation.

Numerical solution required:

1. Assume a FS and compute the FS.
2. Use the computed value as the new assumption and repeat the process.
3. Solution should converge at two decimal places in a few iterations.

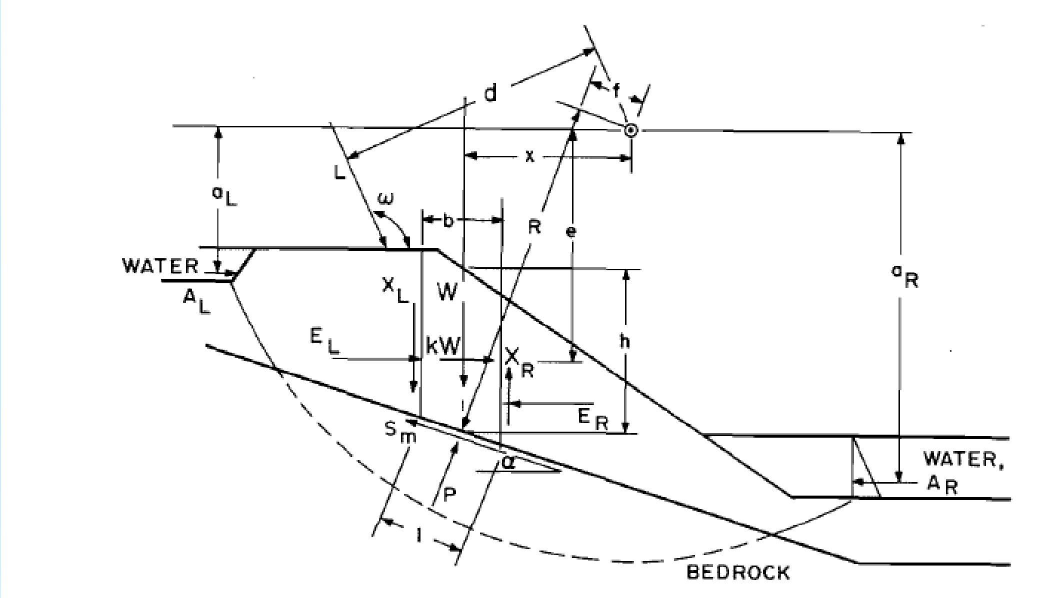
Sliding Blocks Analysis

- Two- or three-part wedge
- Surface replicates some realistic situations
- Inclination of interwedge force is assumed
- Only force equilibrium is solved



Janbu Simplified Method

- Applicable to any shape slip surface
- Does NOT satisfy moment equilibrium
- Ignores shear within the sliding mass, uses correction factor to account for it
- Correction factor, f_o, accounts for soil strength
- Satisfies horizontal force equilibrium
- $FS = \frac{\sum c'l \cos\alpha + (P - ul)\tan\phi' \sin\alpha}{\sum P sin\alpha + \sum kW \pm A - L cos\omega k}$



Summary of Simplified LE Methods

Method	Failure Surface	Soil	Satisfy LE	Comments
ϕ - circle	***Circular***	***Uniform***	***~Yes***	*Serves as the basis for Taylor chart*
Ordinary	***Circular***	***Layered***	***No***	*Can be overly conservative.*
Bishop	***Circular***	***Layered***	***No***	*Use when circular failure is likely*
Wedge	***Planar segments***	***Layered***	***No***	*Use when 2- or 3-part wedge failure is likely*
Simplified Janbu	***General shape***	***Layered***	***No***	*Results may need to be corrected*

Rigorous LE

- We observed that circular slip surface may lead to more critical conditions than planar surface

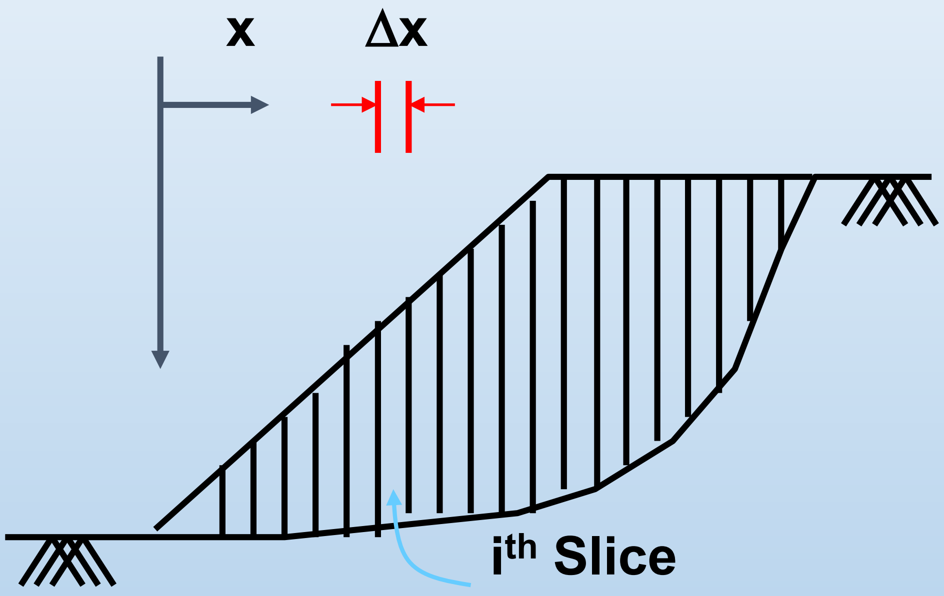
Is there another mechanism that would be still more critical?

- ‘Yes’ when planes of weakness exist or when abrupt changes in strata strength occurs

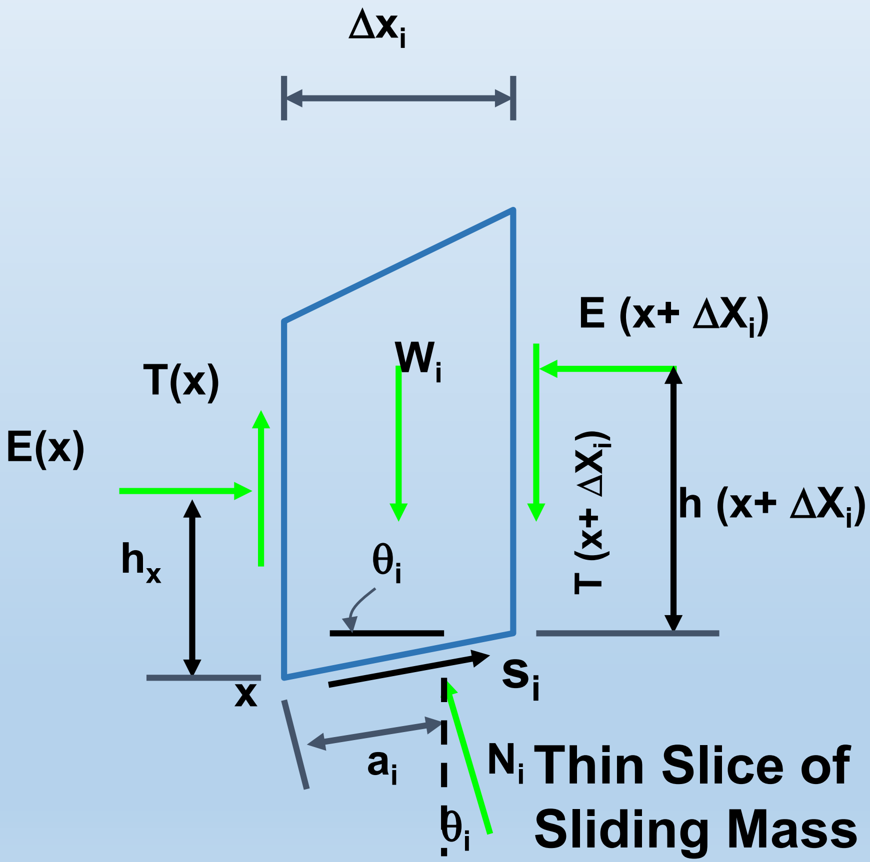
Generalized Method of Slices

- It is therefore worthwhile to consider a generalized slip surface and formulation

Division of General Sliding Mass Into Slices



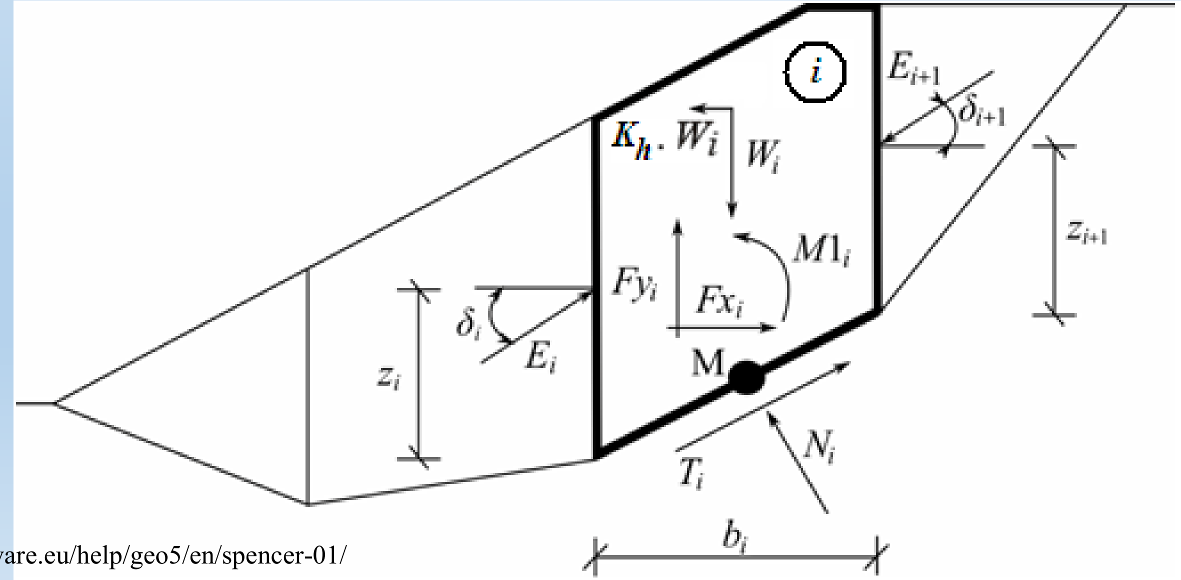
Sliding Mass



Thin Slice of Sliding Mass

Spencer Method

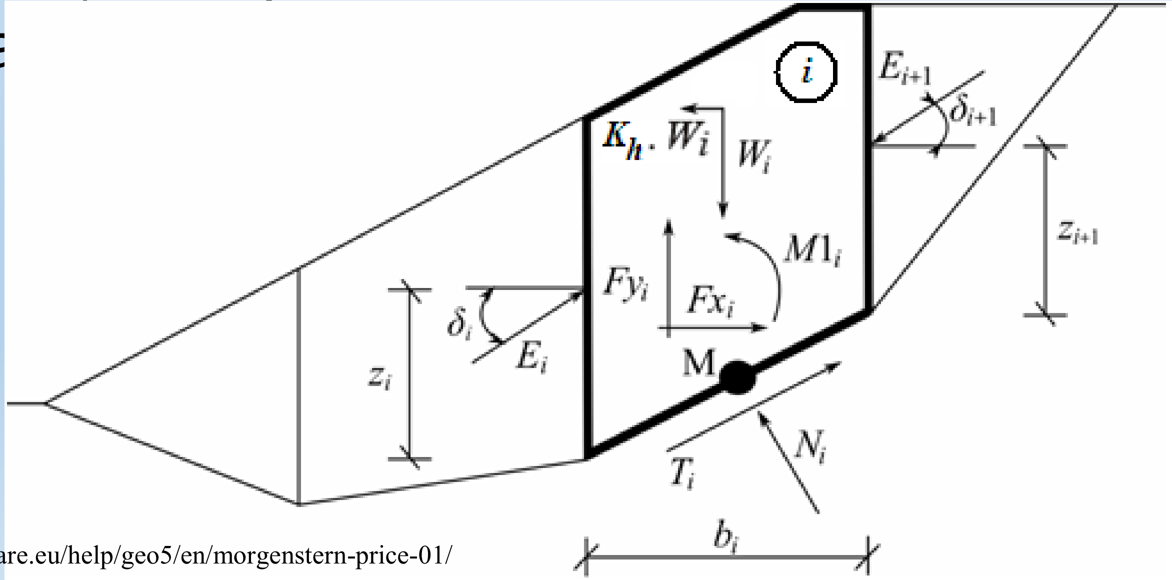
- Rigorous Method, Generalized Slip Surface
- Force and moment assumptions:
  - dividing planes between assumed blocks are always vertical
  - the line of action of weight of block passes through the assumed point of rotation, **M**
  - the normal force N_i is acting at point **M**
  - inclination of forces E_i acting between blocks is constant for all blocks and equals to δ, only at slip surface end points is $\delta = 0$



http://www.finesoftware.eu/help/geo5/en/spencer-01/

Morganstern-Price Method

- Rigorous Method, Generalized Slip Surface
- Same set of forces as Spencer Method. Force and moment assumptions:
  - dividing planes between assumed blocks are always vertical
  - the line of action of weight of block passes through the assumed point of rotation, **M**
  - the normal force N_i is acting at point **M**
  - inclination of interslice forces E_i acting between blocks is different on each block (δ_i), a

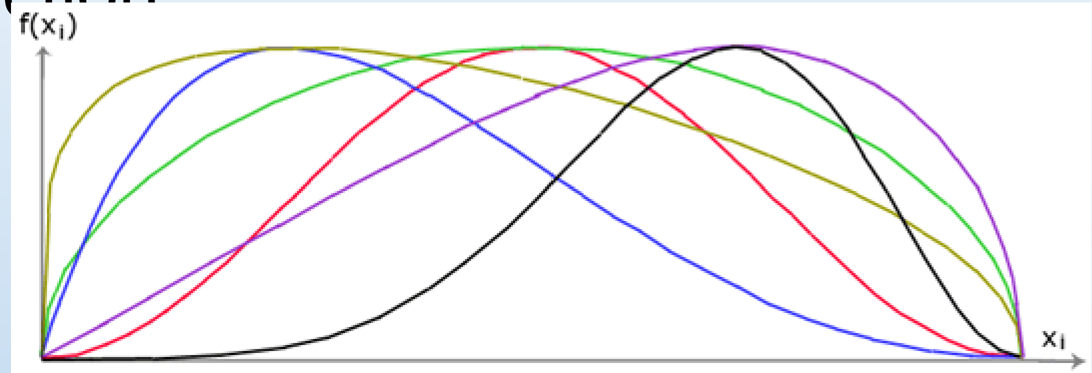


http://www.finesoftware.eu/help/geo5/en/morgenstern-price-01/

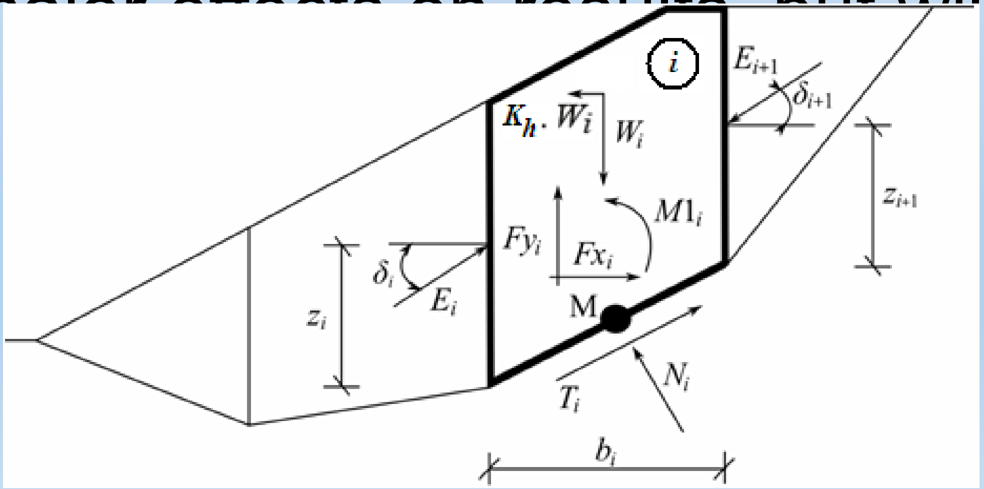
Morganstern-Price Method

- How do we find interslice angle force, δ?
- Need to assume interslice function

$$\delta_i = \lambda_* f(x_i)$$



- Typically a half-sine function
- May not have major effects on results, but will effect convergence



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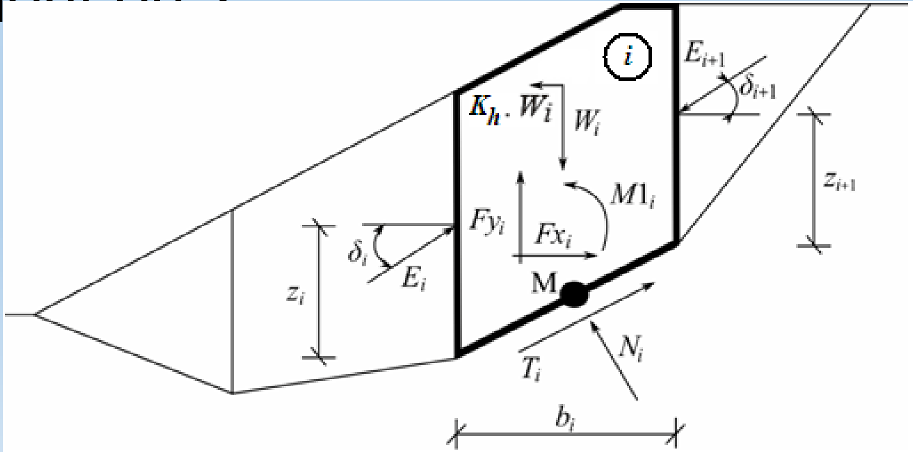
Janbu Method

- Rigorous Method, Generalized Slip Surface
- Same set of forces as Spencer Method. Force and moment assumptions:
  - dividing planes between assumed blocks are always vertical
  - the line of action of weight of block passes through the assumed point of rotation, **M**
  - the normal force N_i is acting at point **M**
  - position z_i of forces E_i acting between blocks is assumed, at slip surface end points is $z = 0$
  - Orientation of interslice shear dependent on z

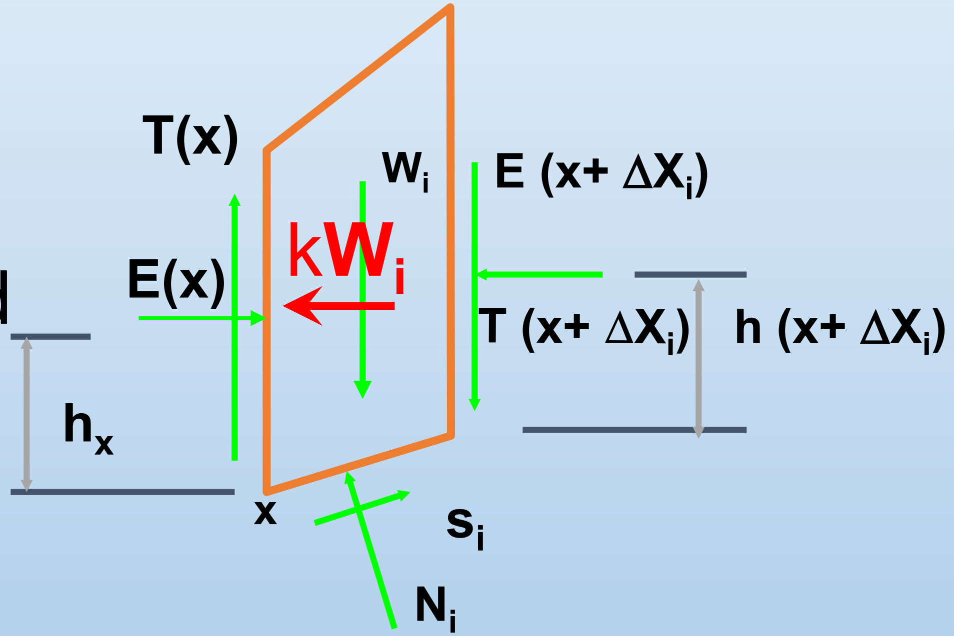
$$\delta_{i+1} = \arctan\left(\frac{2.z_{i+1}}{b_i} + \tan\alpha_i\right) - \arcsin\frac{E_i\left(\cos\delta_i\left(z_i - \frac{b_i.\tan\alpha_i}{2}\right) + \sin\delta_i.\frac{b_i}{2}\right) - M1_i}{E_{i+1}\sqrt{\left(z_{i+1} + \frac{b_i.\tan\alpha_i}{2}\right)^2 + \left(\frac{b_i}{2}\right)^2}}$$



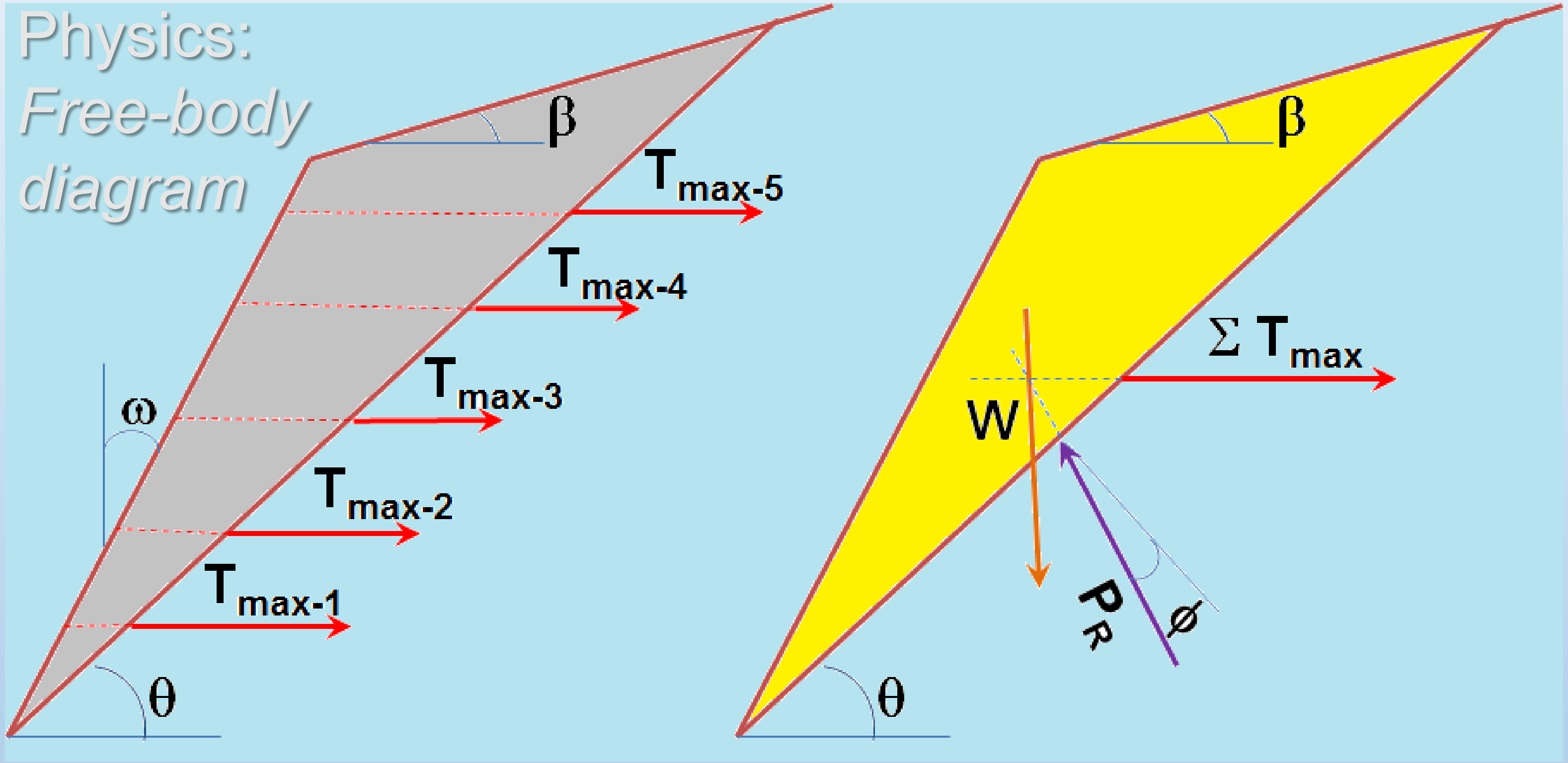
http://www.finesoftware.eu/help/geo5/en/janbu-01/

Seismic Stability using LE Analysis

- Pseudostatic force represents earthquake
- Inertia force is horizontal and equal to k•W
- k represents the design horizontal acceleration divided by g
- k is typically 0.5 of maximum ground acceleration
- Approach can be implemented in any analysis



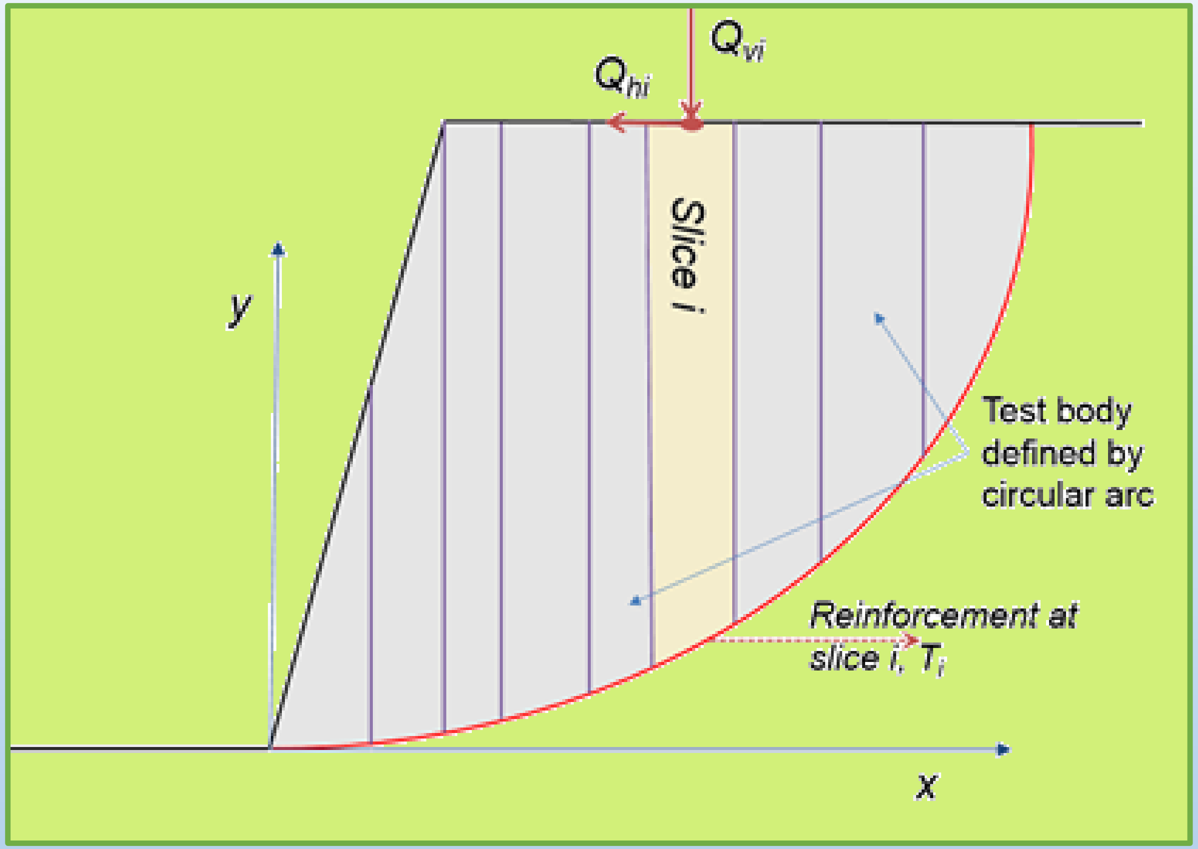
Reinforced Soil: Culmann (1866)

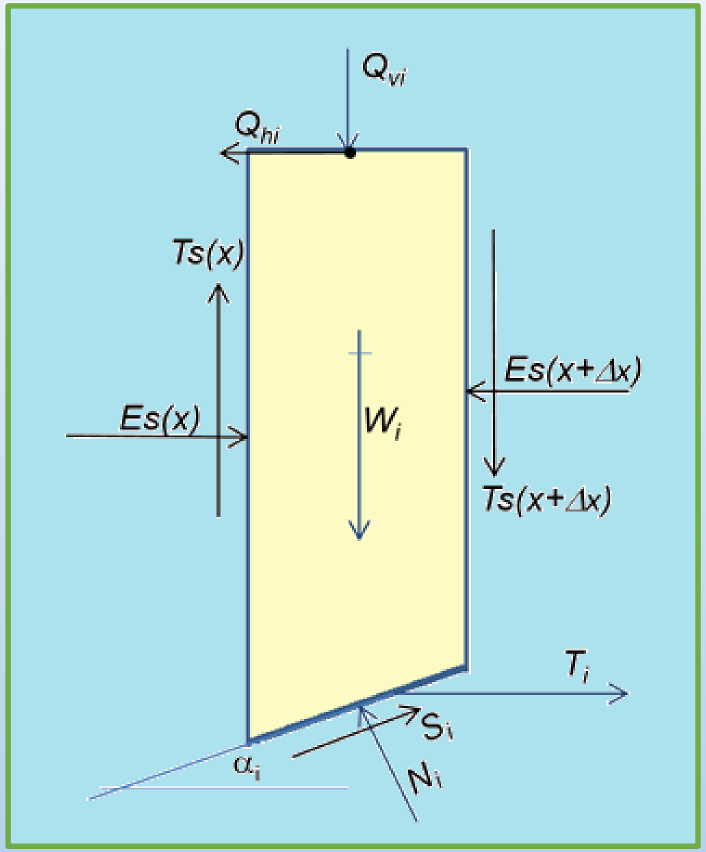


Small stretch of reinforcement ➔ Active wedge develops ➔ Load in reinforcement drops to T_{max}

Note: *Formation of slip surface does not mean structural failure ➔ Reinforcement is designed to resist the active soil wedge*

Reinforced Soil: Bishop (1955) Circular Arc





Bishop considers layered soil/complex problems. Circle can degenerate to planar surface (if it is more critical) but a priori assumed planar surface cannot degenerate to curved surface ➔ Valid for slopes and walls…

Questions?

Back-Analysis and Forensics

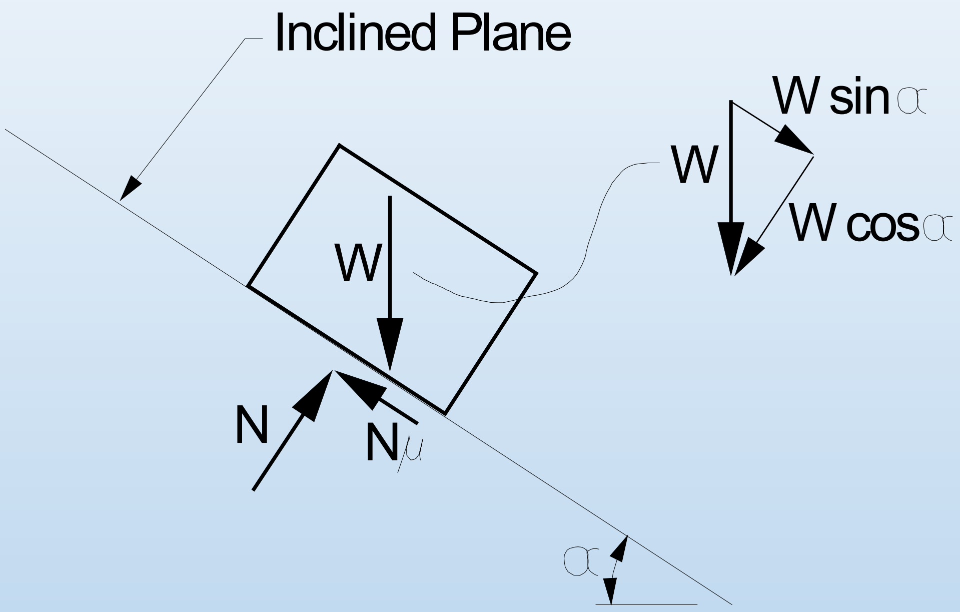
A useful tool for estimating soil strength – prototype scale.

Cases:

- Existing stable slopes and we need to design new stable slopes in the same soil.
- A slope is showing distress, or has failed, and we need to design stabilizing measures

Basic Assumption: The Factor of Safety is known – usually equal to 1.0, but higher values may be appropriate in some cases.

A simple case – a block on an inclined plane.



Sum Forces perpendicular
to the inclined plane.

$$\sum F_{perpendicular} = 0:$$

$$N - W \cdot \cos\alpha = 0$$

Sum Forces parallel
to the inclined plane:

$$\sum F_{parallel} = 0:$$

$$W \cdot \sin\alpha - N \cdot \mu = 0$$

Solving simultaneously:

$$\mu = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$$

The angle, α , is in effect a friction angle.

Let's look at Bishop's Equation:

$$FS = \frac{\sum \left[\frac{c'b + (W - ub)\tan\phi'}{\cos\alpha\left(1 + \frac{\tan\alpha\tan\phi'}{FS}\right)} \right]}{\sum W\sin\alpha}$$

With the Factor of Safety = 1.0, and all elements of slope geometry known [*W, u, b,* and α], there are two unknowns in the equation, *c'* and ϕ'.

One Equation and two unknowns – no can do!

With both the constant and variable portions of strength unknown:

We must either know or assume one in order to solve for the other.

Can we do this?

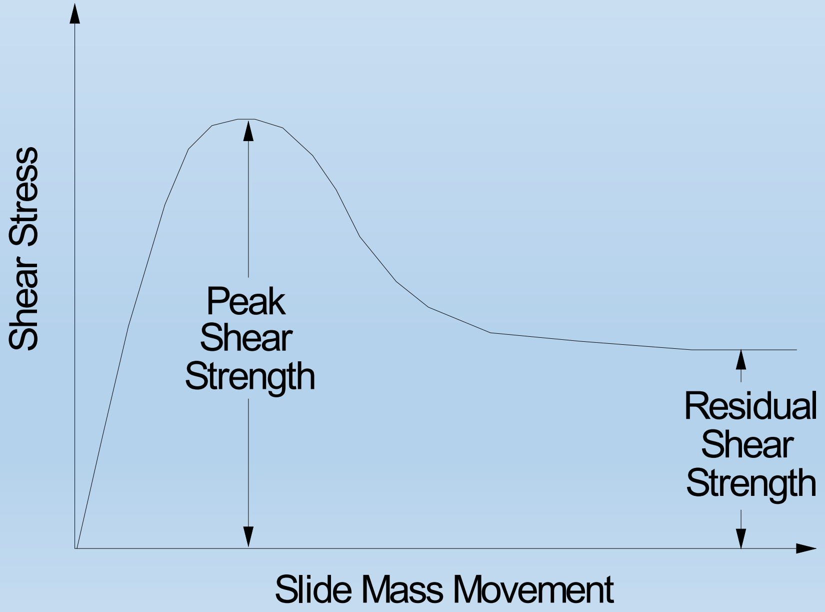
Yes and No

Analyze the situation.

Start with the type of failure.

Was the failure [or near failure] drained or undrained?

- Undrained – only realistic for clay and plastic silt soils that failed or are near failure as a result of a load change.
  - For the condition just prior to failure [original slope geometry] set $\phi' = 0$ and solve for c_u.
  - For the condition just after failure [post failure slope geometry] set $\phi' = 0$ and solve for $(c_u)_{residual}$



Drained – sand slopes or clay slopes long after construction or other loading.

- Sands either before or after failure, set c’ = 0, and solve for ϕ’.
- Soils that could have c’ and ϕ’:
    - Estimate ϕ’ and solve for c’.
    - If the slope is in a failure geometry, set c’ = 0 and solve for ϕ’.
    - Option exists to compute τ_{ff} and use it for shear strength [this is numerically equal to using c_u].

Let’s look at the mathematics:

With c’ = 0, and FS = 1.0 :

$$FS = \frac{\sum \left[\dfrac{c' \cdot b + (W - ub)\tan\phi'}{\cos\alpha \left(1 + \dfrac{\tan\alpha \tan\phi'}{FS} \right)} \right]}{\sum W \sin\alpha}$$

$$\sum W \sin\alpha = \sum \left[\frac{(W - ub)\tan\phi'}{\cos\alpha \left(1 + \tan\alpha \tan\phi' \right)} \right]$$

Must use an iterative approach to obtain a value of φ'

Can use goal seek in excel, so nothing is difficulty about this.

With $\varphi' = 0$, and FS = 1.0:

$$FS = \frac{\sum \left[\frac{c' \cdot b + (W - ub)\tan\phi'}{\cos\alpha \left(1 + \frac{\tan\alpha \tan\phi'}{FS} \right)} \right]}{\sum W \sin\alpha}$$

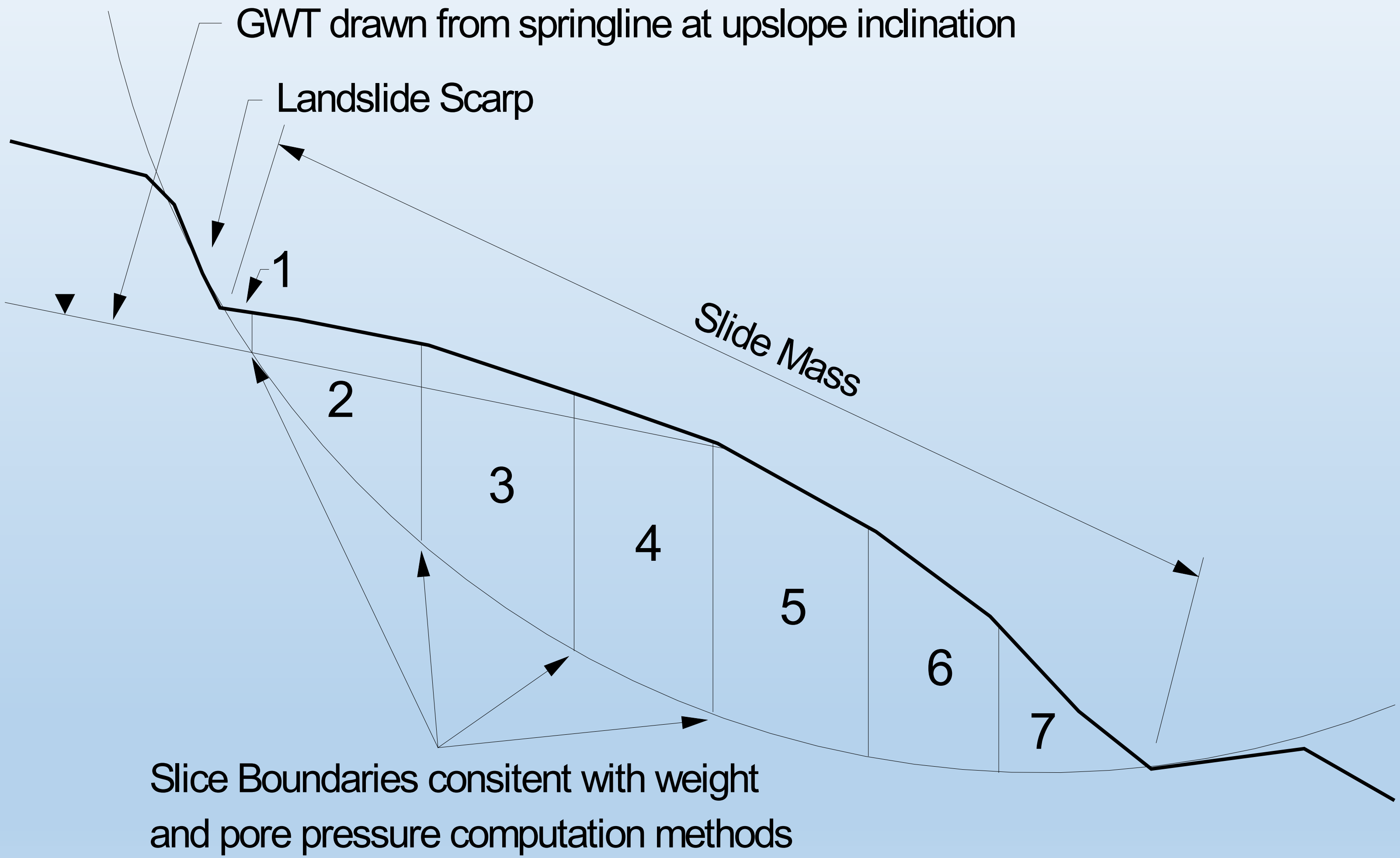
$$\sum W \sin\alpha = \sum \left[\frac{c_u \cdot b}{\cos\alpha} \right]$$

Since c_u is a constant, it can be moved outside the summation,

$$c_u = \frac{\sum W \sin\alpha}{\sum \frac{b}{\cos\alpha}}$$

A closed form solution.

Let's take an example >> Do the Back Analysis.



Let’s consider some postulated conditions.

- The slide occurred in the middle of winter a considerable length of time after construction – months anyway.
- Hence the failure was a drained failure that occurred because of high gravitational seepage derived pore water pressures during the wet season.
- But, now the slide mass has moved.
  - During the movement, the total stresses were changing simply because the slide geometry was changing, and the soil strength was being reduced because of the shearing action along the failure surface.
  - So, when the slide came to rest, there were likely excess pore pressures present – undrained.

But,

- When we get around to stabilizing the slide mass – we'll use your design for that – it will be months after the failure, hence it is likely that any excess pore pressures will have dissipated, and the slope will be in the drained state again.
- Pore pressures will again be only the result of gravitational seepage.
- So, we should do the back analysis for the drained case with gravitational pore pressures.
    - Just to keep things manageable, let's all use hydrostatic pore pressures – that way we don't need to draw a flow net, we only have to have the GWT.

We still have some choices to make:

- For the drained case, we want to have a value for φ´, but should we set c’ = 0 or not.
  - Options:
    - Assume a value for c’ and solve for φ´.
    - Assume a value for φ´, and solve for c’.
    - Set c’ = 0 and solve for φ´.

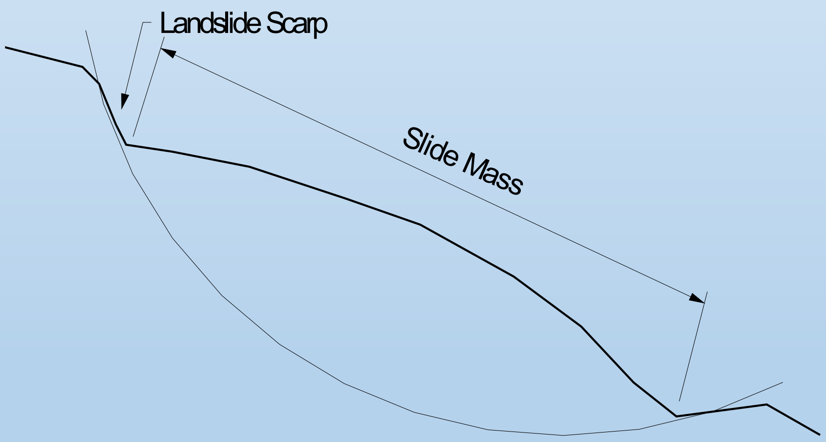
Two answers:

1. The failure and the shearing the goes with it remolds the soil, thus “destroying” the “cohesion intercept” of the strength relationship.
2. We can also do a reasonably good job of estimating the soil friction angle, hence we could assume φ´, and solve for c’.

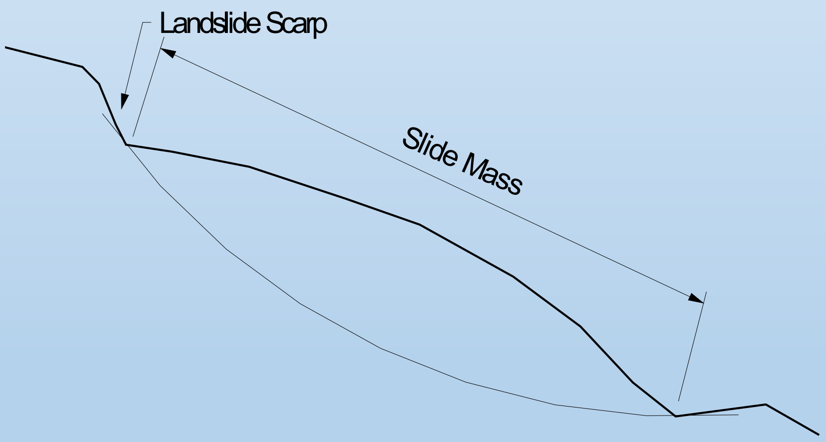
Let’s all agree in this case to set c’ = 0 and solve for φ′.

What range in values of φ′ might we expect from our back analysis?

It depends on the exact shape of the circular failure surface that we use.



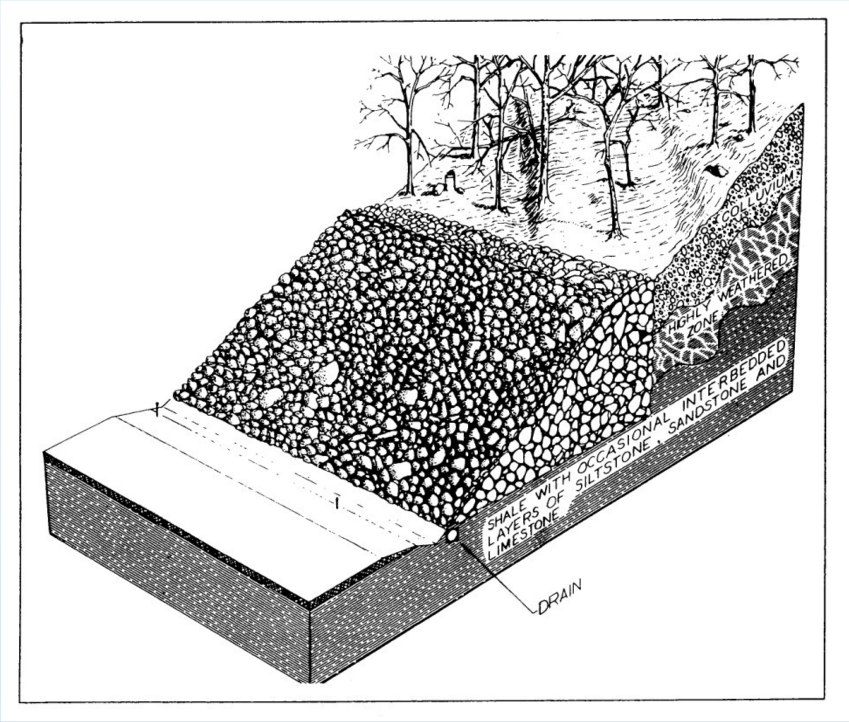
A “deep” failure surface will produce a lower φ′ angle.



A “shallow” failure surface will produce a much higher φ′ angle.

These differences will largely compensate in the buttress design, so that the end result will not be dramatically different.

So, how do we design a rock buttress?



Step 1. Select a Trial Size.

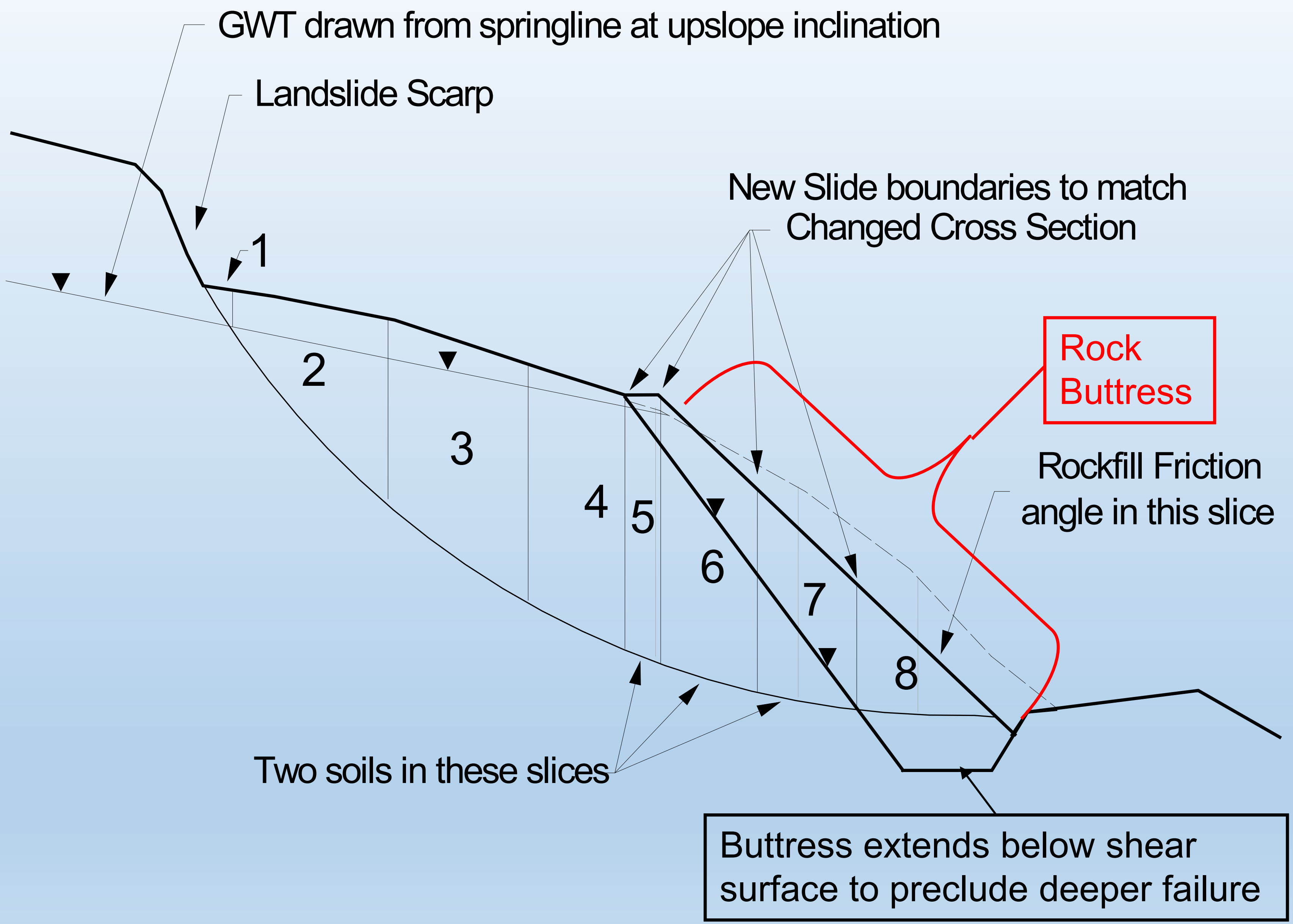
Step 2. Analyze the trial buttress for stability.

Step 3. If FS = 1.3, design is OKAY.

If FS < 1.3, increase buttress size.

If FS > 1.3 consider reducing buttress size.

DESIGN IS A TRIAL AND ERROR PROCESS!


GWT drawn from springline at upslope inclination
Landslide Scarp
New Slide boundaries to match
Changed Cross Section
Rock
Buttress
Rockfill Friction
angle in this slice
1
2
3
4
5
6
7
8
Two soils in these slices
Buttress extends below shear
surface to preclude deeper failure

Design Notes

- Width of Buttress controls the Factor of Safety.
- Rockfill is so pervious that the groundwater simply runs down the back face of the buttress – where does it go?
    - Better provide drainage.
- The Buttress should extend below the failure surface to insure that a failure surface does not form under the buttress > ≈ 3 ft.
- If you use a "spreadsheet", you will need to modify it to handle slice with two soils, or manually compute the slice weights. You will also need to deal with the strengths of the two different soils
- New slice boundaries may require more slices, hence you will need to add to a "Spreadsheet", and make sure that the Factor of Safety summation is correct.
- At some point in the design process you should be thinking about how the buttress will be constructed.

Practical aspects of Construction

- How will the excavation be done?
  - Dozer
  - Track-hoe
- How will the rock fill be placed?
  - Dumped – this requires a truck ramp and a minimum width of something like 10 ft to 12 ft – maybe a bit more.
  - Placed with a track-hoe with a thumb bucket - How far … machine